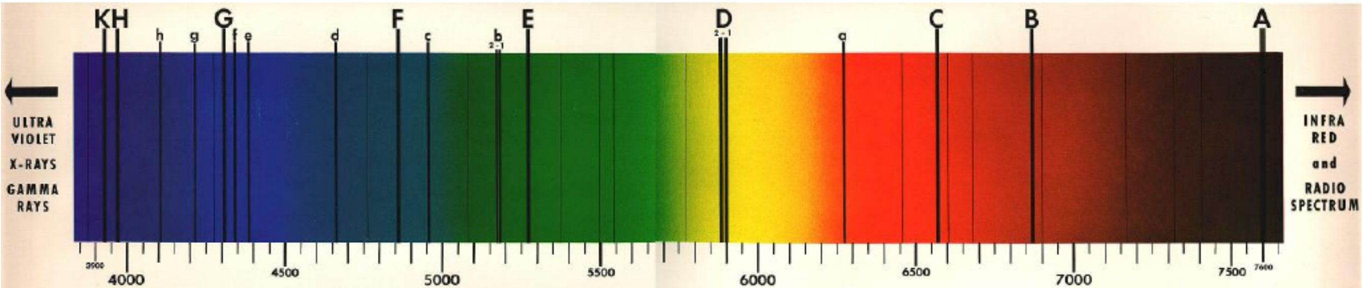
STELLAR/SOLAR SPECTRA EXERCISES

Fraunhofer's solar spectrum



Wikipedia: *Kirchhoff's three laws of spectroscopy:*

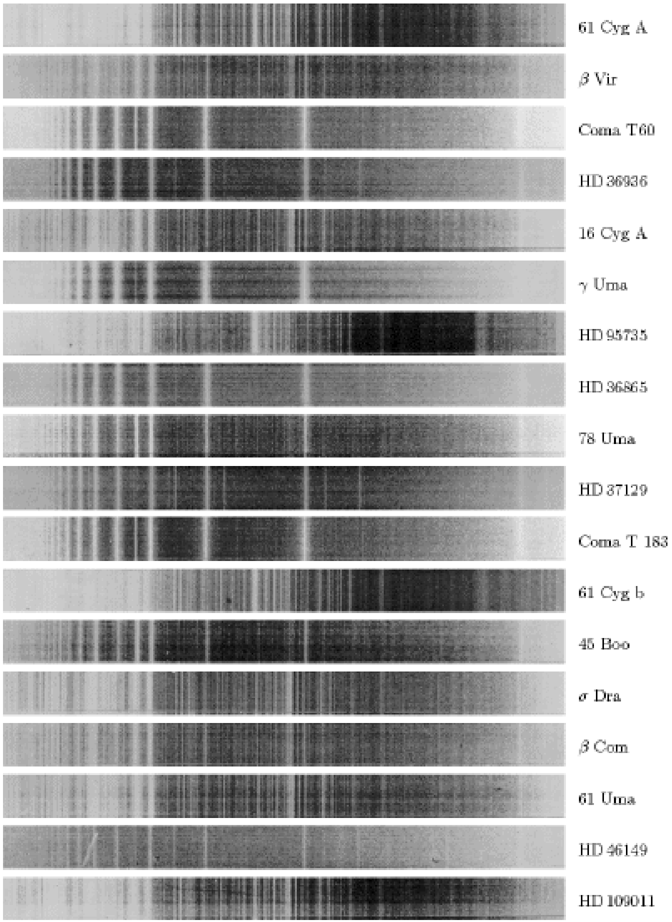
1. A hot solid object produces light with a continuous spectrum.

2. A hot tenuous gas produces light at specific, discrete colors which depend on properties of the elements in the gas.

3. A hot solid object surrounded by a cool tenuous gas produces light with an almost continuous spectrum which has gaps at specific, discrete colors depending on properties of the elements in the gas.

Kirchhoff did not know about the existence of energy levels in atoms. The existence of discrete spectral lines was later explained by the Bohr model of the atom, which helped lead to quantum mechanics.

Much spectral variation between stars – is there order?

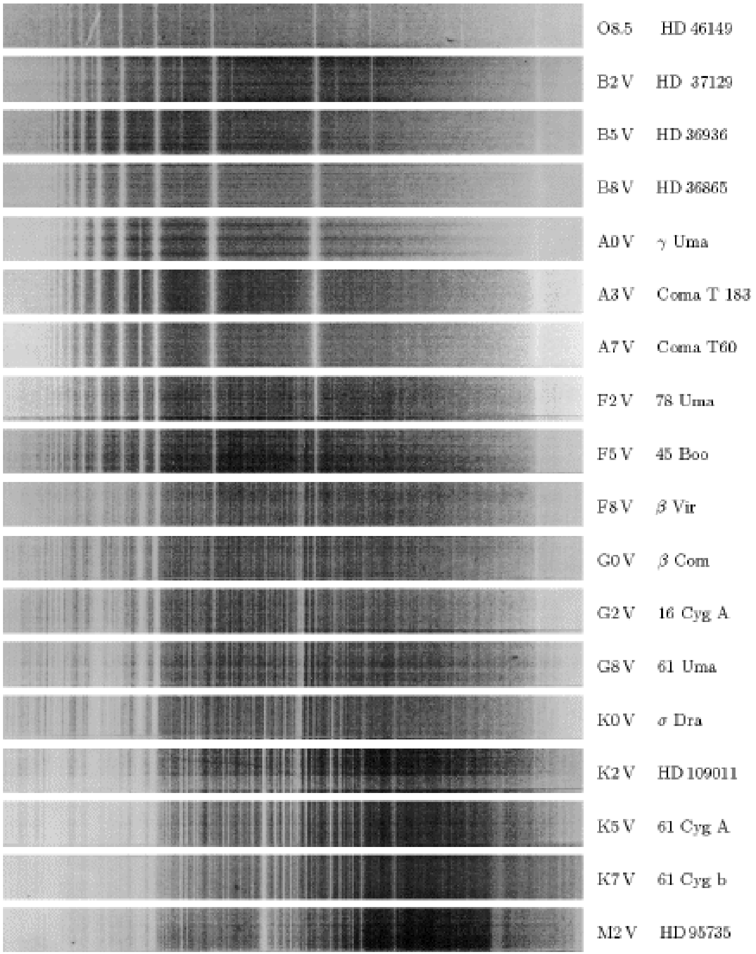


Pickering's harem

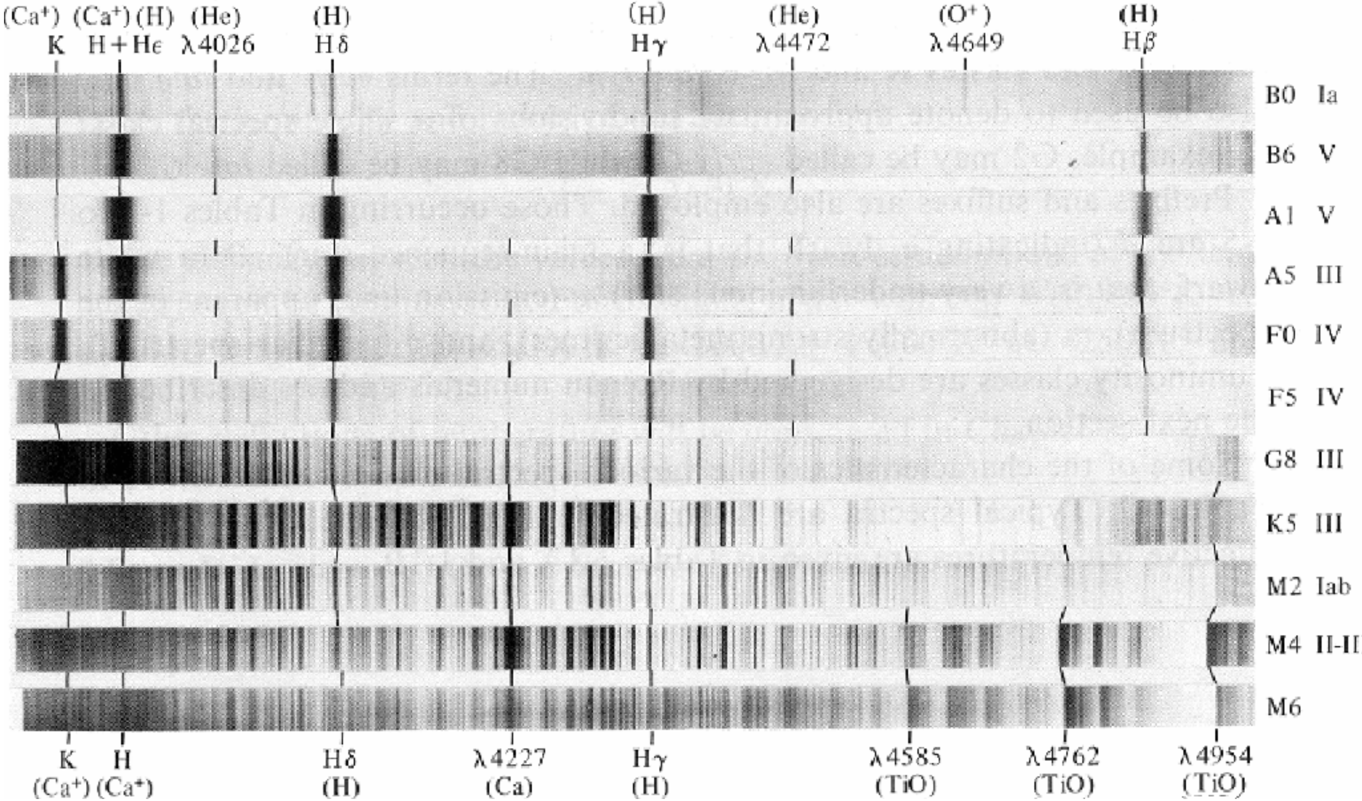
Wkipedia: *Edward Charles Pickering (director of the Harvard Observatory from 1877 to 1919) decided to hire women as unskilled workers to process astronomical data. Among these women were Williamina Fleming, Annie Jump Cannon, Henrietta Swan Leavitt and Antonia Maury. This staff came to be known as "Pickering's Harem" or, more respectfully, as the Harvard Computers.*

Annie Cannon

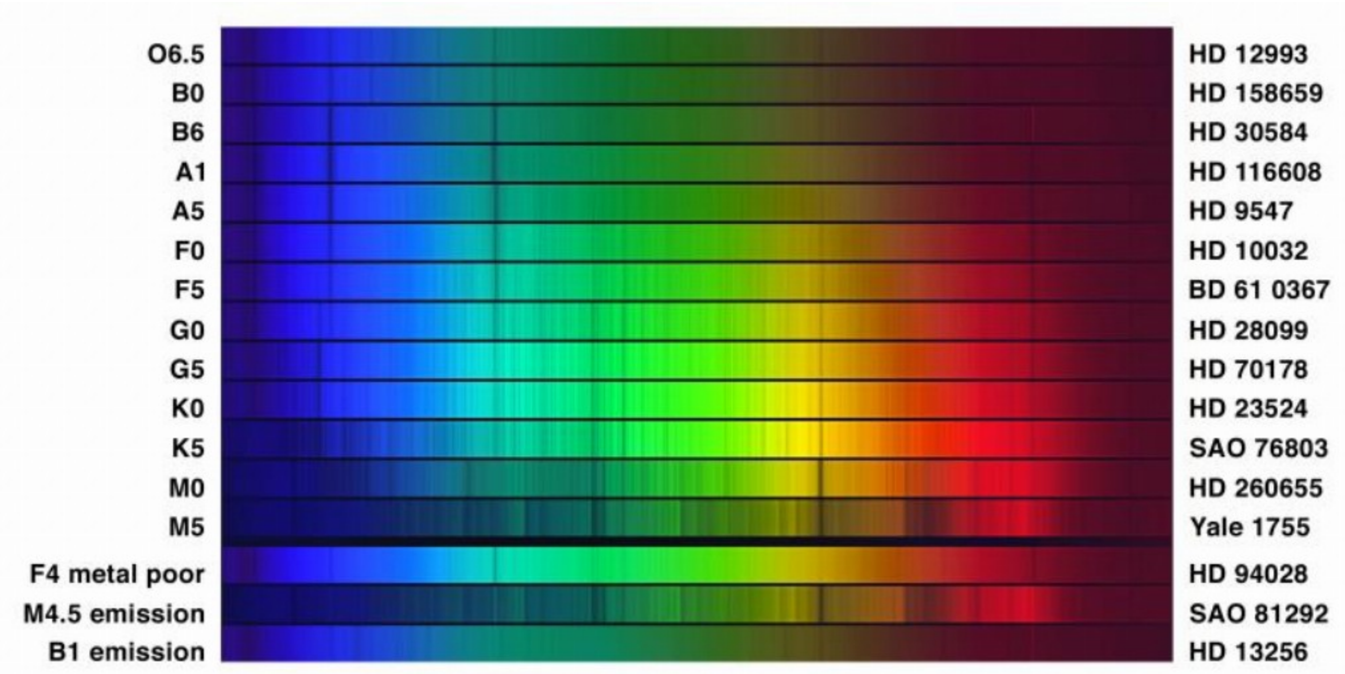
Main-sequence stellar spectra ordered



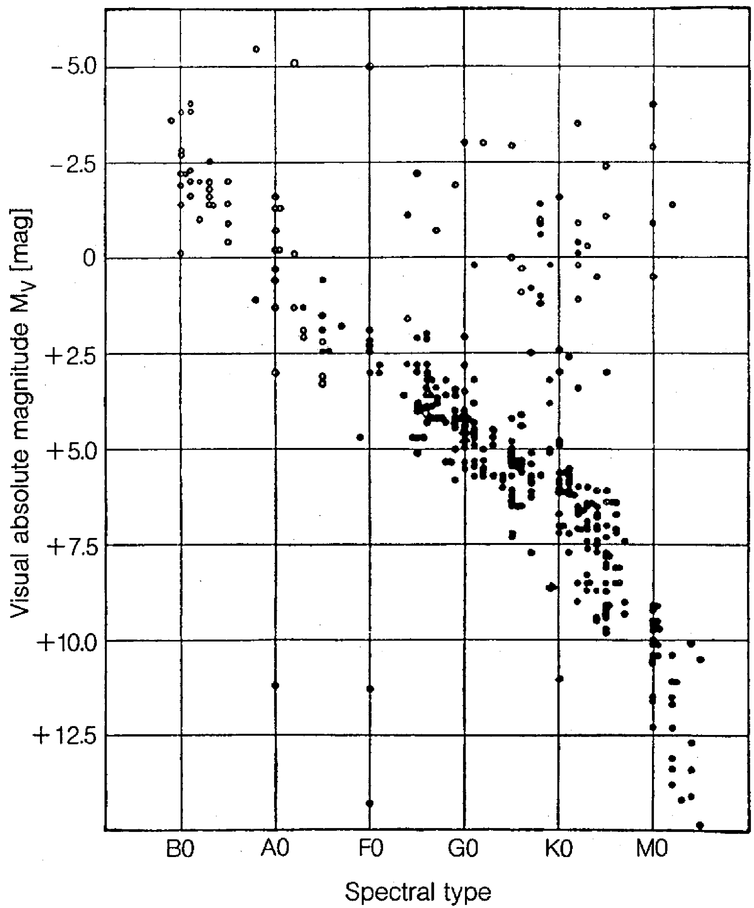
Harvard classification



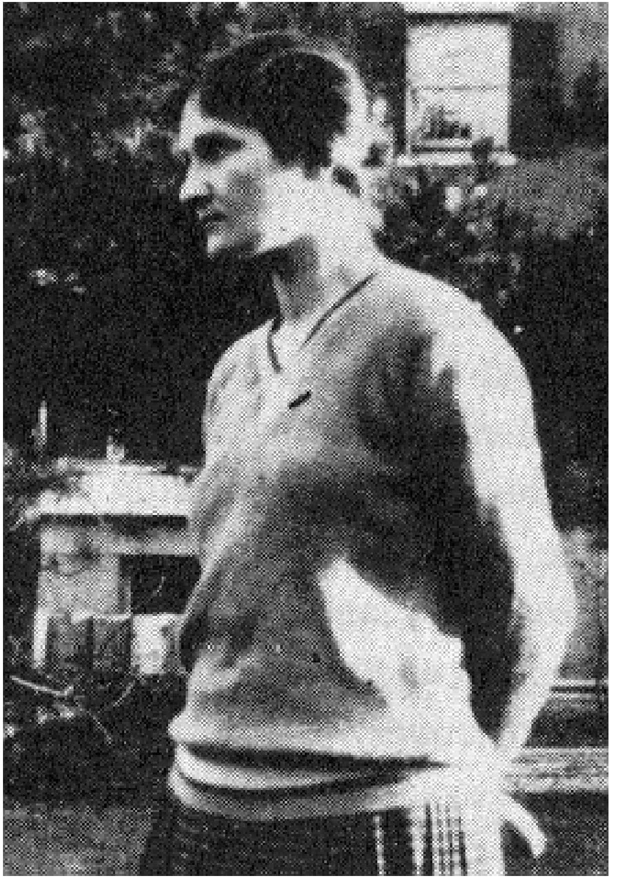
Harvard classification



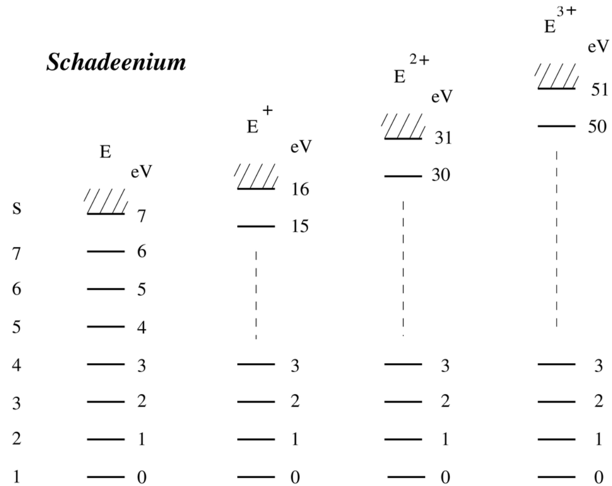
Empirical Hertzsprung-Russell diagram



Cecilia Payne



Saha-Boltzmann equations



Boltzmann distribution per ionization stage: $\frac{n_{r,s}}{N_r} = \frac{g_{r,s}}{U_r}\,\mathrm{e}^{-\chi_{r,s}/kT}$

partition function: $U_r \equiv \sum_s g_{r,s}\,\mathrm{e}^{-\chi_{r,s}/kT}$

Saha distribution over ionization stages:

$$\frac{N_{r+1}}{N_r} = \frac{1}{N_\mathrm{e}} \frac{2\,U_{r+1}}{U_r} \left(\frac{2\pi m_\mathrm{e} kT}{h^2}\right)^{3/2} \mathrm{e}^{-\chi_r/kT}$$

Schadeenium Saha-Boltzmann populations

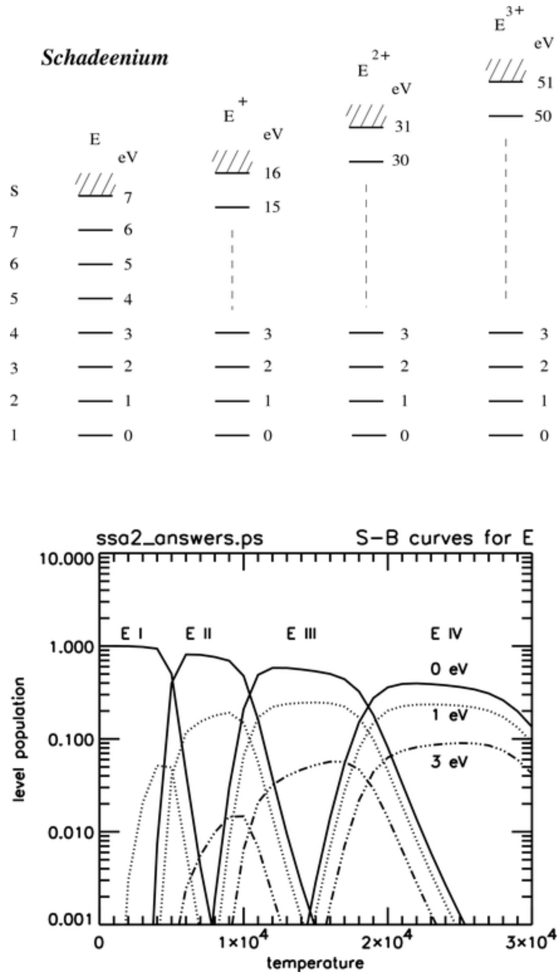
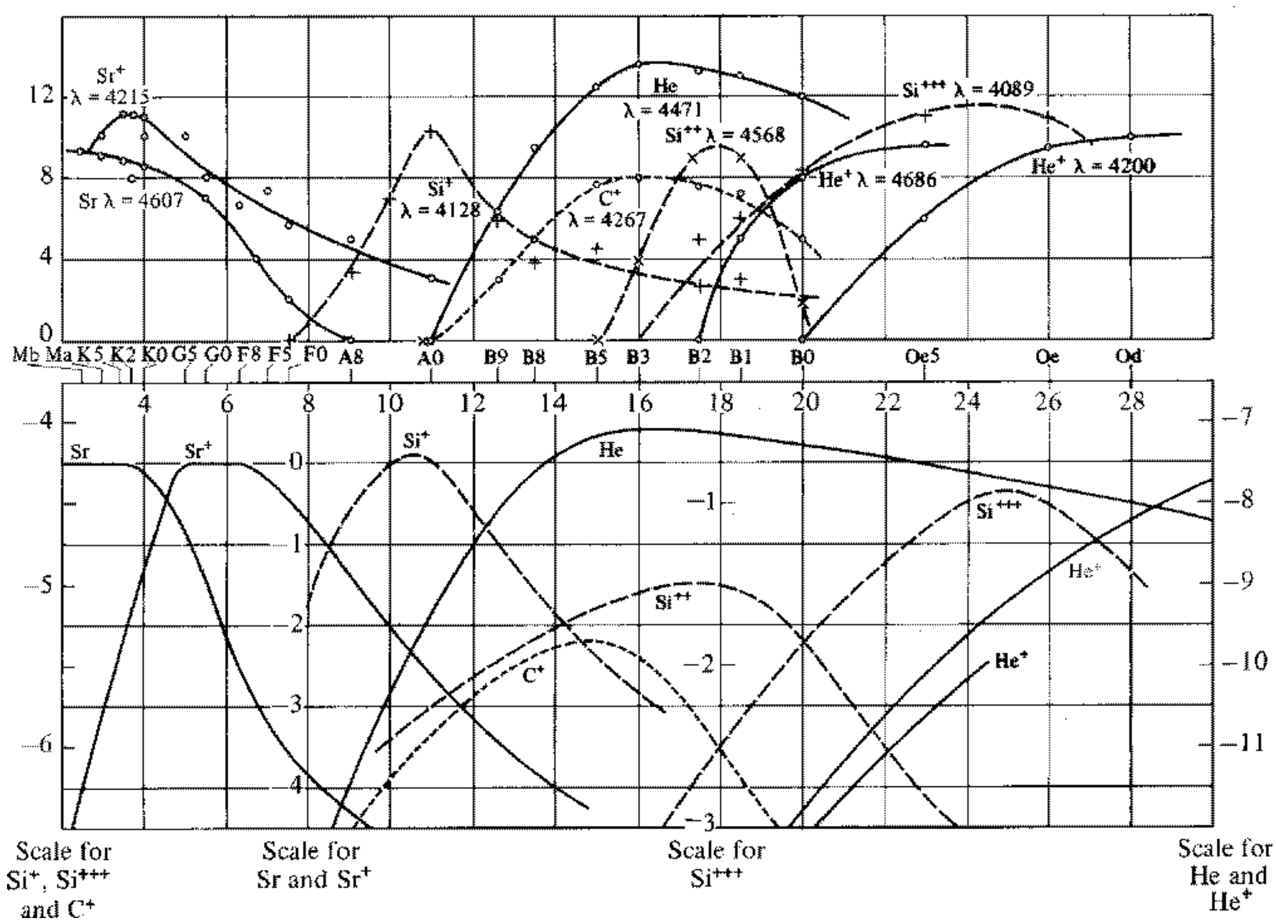
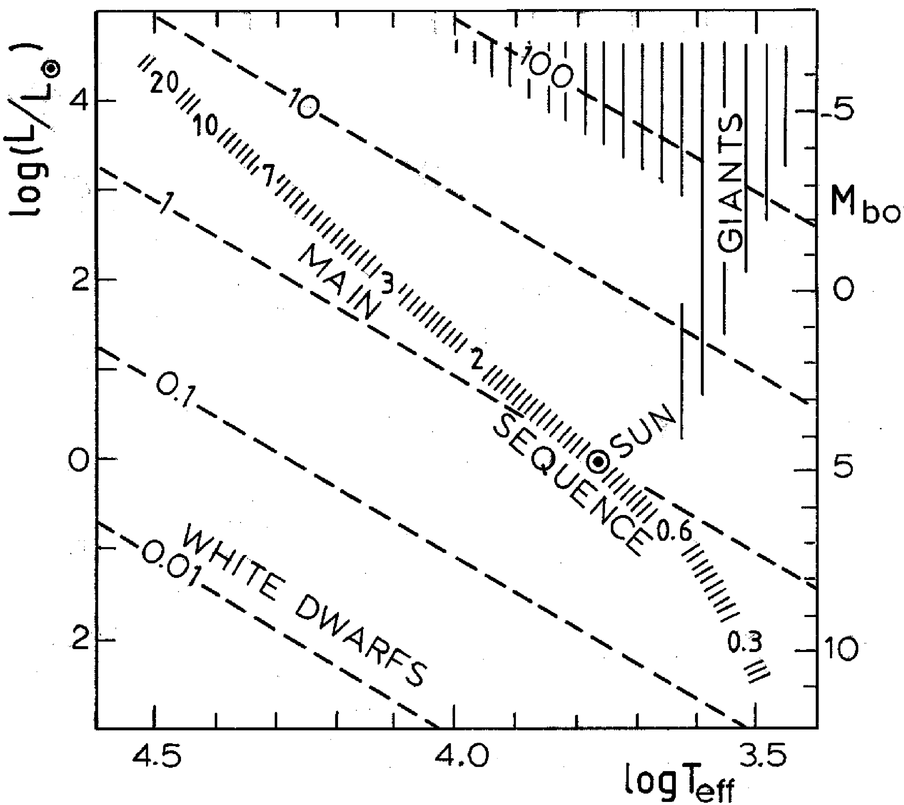


Figure 2.8: Saha-Boltzmann distributions for schadeenium, a didactic element not unlike iron invented by Aert Schadee, with symbol E. *Upper diagram*: Energy level diagram. The level energies increase in 1 eV steps. The columns may be thought stacked on top of each other since each ion requires the previous stage to be ionized. In astronomical convention the spectra of neutral schadeenium E and once-ionized schadeenium E^+ are called E I, E II, etc. *Lower diagram*: Saha-Boltzmann population fractions for levels 1, 2 and 4 of stages E I – E IV as function of temperature. All statistical weights $g_{r,s}$ were assumed unity. The population of an excited level increases with temperature until its stage ionizes. Only two stages co-exist effectively at any temperature. From my second "Stellar Spectra A" exercise at `http://www.astro.uu.nl/~rutten`. Aert Schadee (1936 – 1999) was an astrophysicist at Utrecht.

Cannon's classification and Payne's Saha-Boltzmann curves

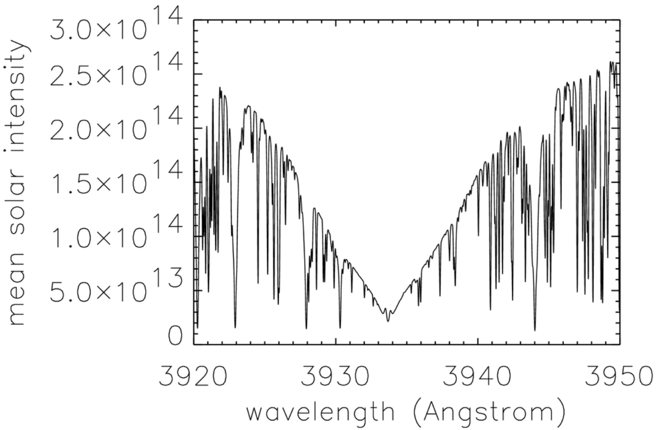


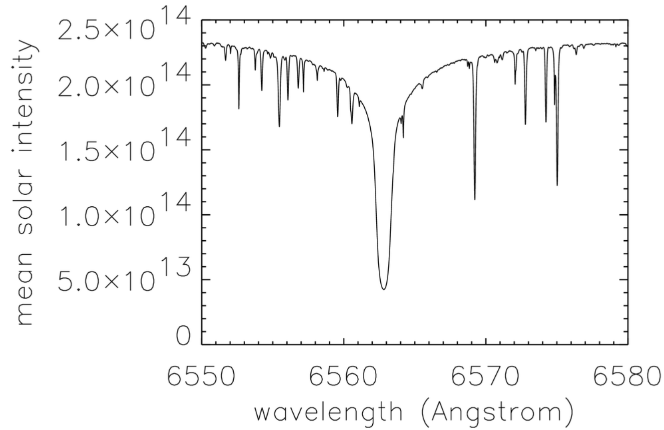
Physical Hertzsprung-Russell diagram



$$L = 4\pi R^2\,\sigma T_{\rm eff}^4 \tag{1}$$

Ca II K and Hα in the photospheric spectrum

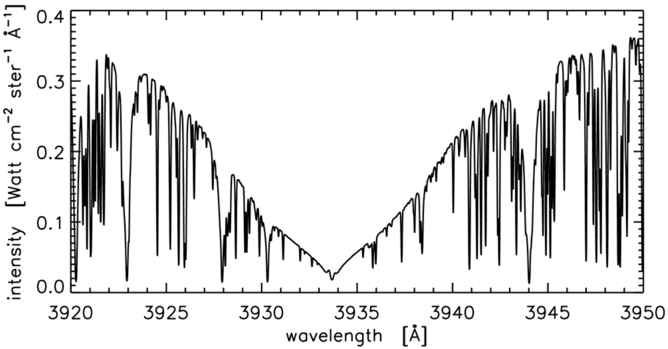


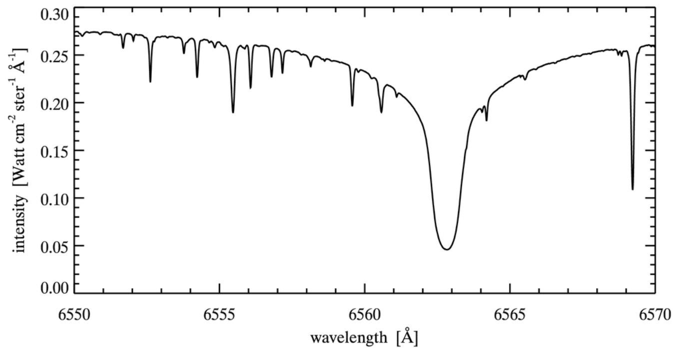


Solar abundances and ionization energies (eV)

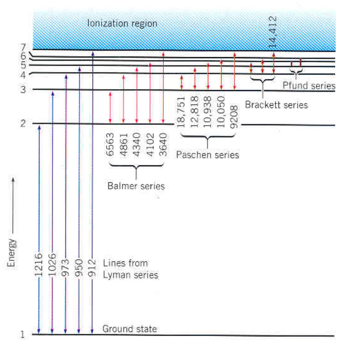
nr.	element	solar abundance	χ_1	χ_2	χ_3	χ_4
1	H	1	13.598	–	–	–
2	He	7.9×10^{-2}	24.587	54.416	–	–
6	C	3.2×10^{-4}	11.260	24.383	47.887	64.492
7	N	1.0×10^{-4}	14.534	29.601	47.448	77.472
8	O	6.3×10^{-4}	13.618	35.117	54.934	77.413
11	Na	2.0×10^{-6}	5.139	47.286	71.64	98.91
12	Mg	2.5×10^{-5}	7.646	15.035	80.143	109.31
13	Al	2.5×10^{-6}	5.986	18.826	28.448	119.99
14	Si	3.2×10^{-5}	8.151	16.345	33.492	45.141
20	Ca	2.0×10^{-6}	6.113	11.871	50.91	67.15
26	Fe	3.2×10^{-5}	7.870	16.16	30.651	54.8
38	Sr	7.1×10^{-10}	5.695	11.030	43.6	57

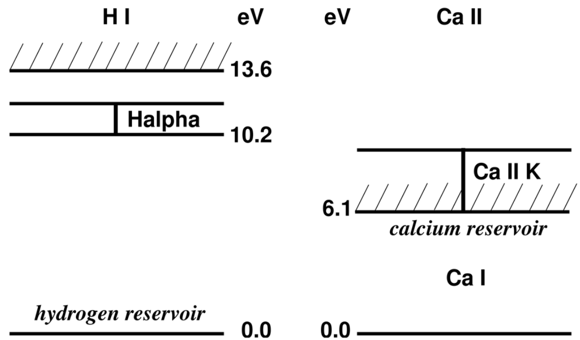
Ca II K and Hα in the photospheric spectrum

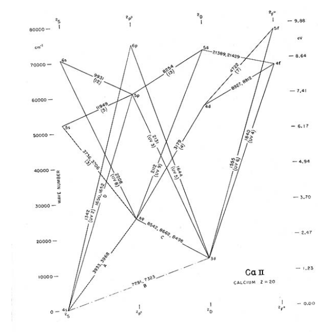




solar abundances: Ca/H = 2×10^{-6}







Assuming LTE at $T = 5000$ K, $P_e = 10^2$ dyne cm^{-2}:

Boltzmann H I: $\frac{n_2}{n_1} = 4.2 \times 10^{-10}$ Saha Ca II: $\frac{N_{\mathrm{Ca\,II}}}{N_{\mathrm{Ca}}} \approx 1$ $\frac{\mathrm{Ca\,II}\,(n=1)}{\mathrm{H\,I}\,(n=2)} = 8 \times 10^3$

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