SOLAR SPECTRUM FORMATION: THEORY

Robert J. Rutten
https://webspace.science.uu.nl/~rutte101

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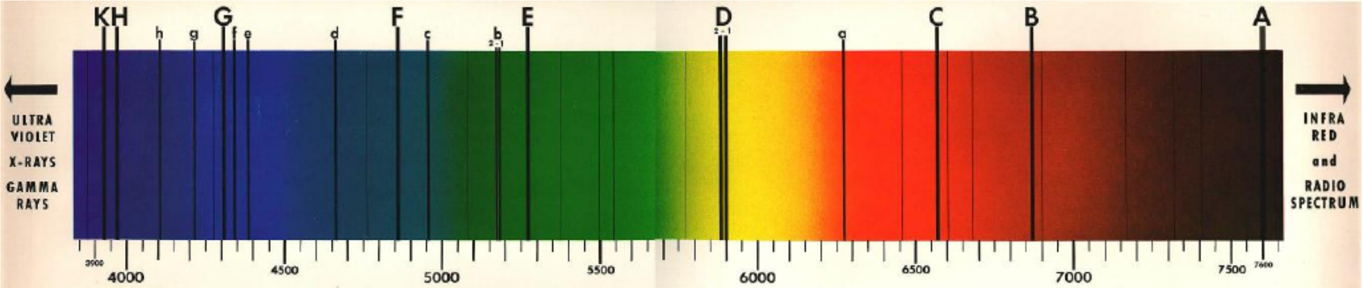
Oh Be A Fine Girl

Fraunhofer's solar spectrum



Wikipedia: *Kirchhoff's three laws of spectroscopy:*

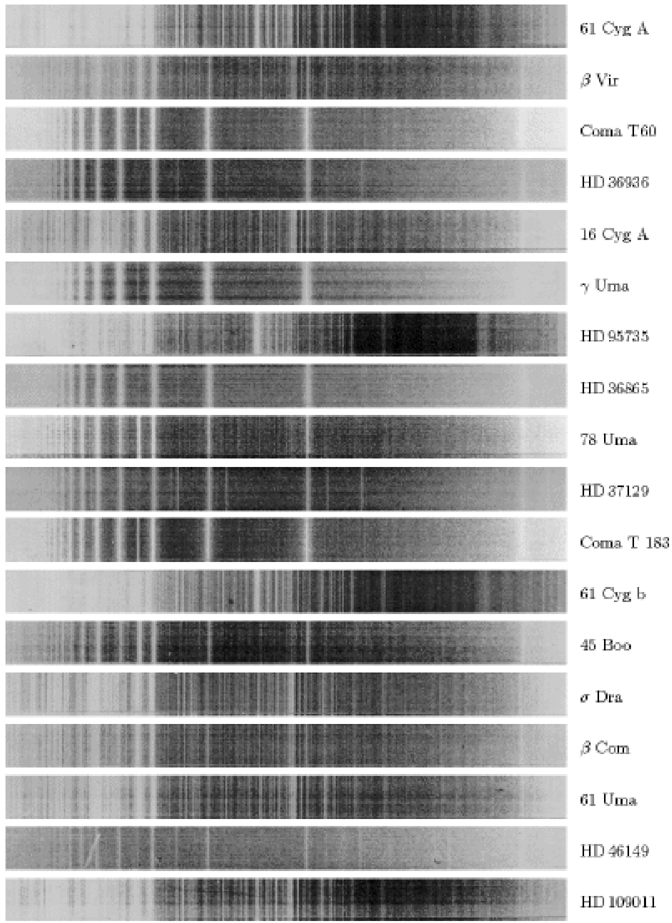
1. A hot solid object produces light with a continuous spectrum.

2. A hot tenuous gas produces light at specific, discrete colors which depend on properties of the elements in the gas.

3. A hot solid object surrounded by a cool tenuous gas produces light with an almost continuous spectrum which has gaps at specific, discrete colors depending on properties of the elements in the gas.

Kirchhoff did not know about the existence of energy levels in atoms. The existence of discrete spectral lines was later explained by the Bohr model of the atom, which helped lead to quantum mechanics.

Much spectral variation between stars – is there order?



Pickering's harem

Wkipedia: *Edward Charles Pickering (director of the Harvard Observatory from 1877 to 1919) decided to hire women as unskilled workers to process astronomical data. Among these women were Williamina Fleming, Annie Jump Cannon, Henrietta Swan Leavitt and Antonia Maury. This staff came to be known as "Pickering's Harem" or, more respectfully, as the Harvard Computers.*

Harvard classification

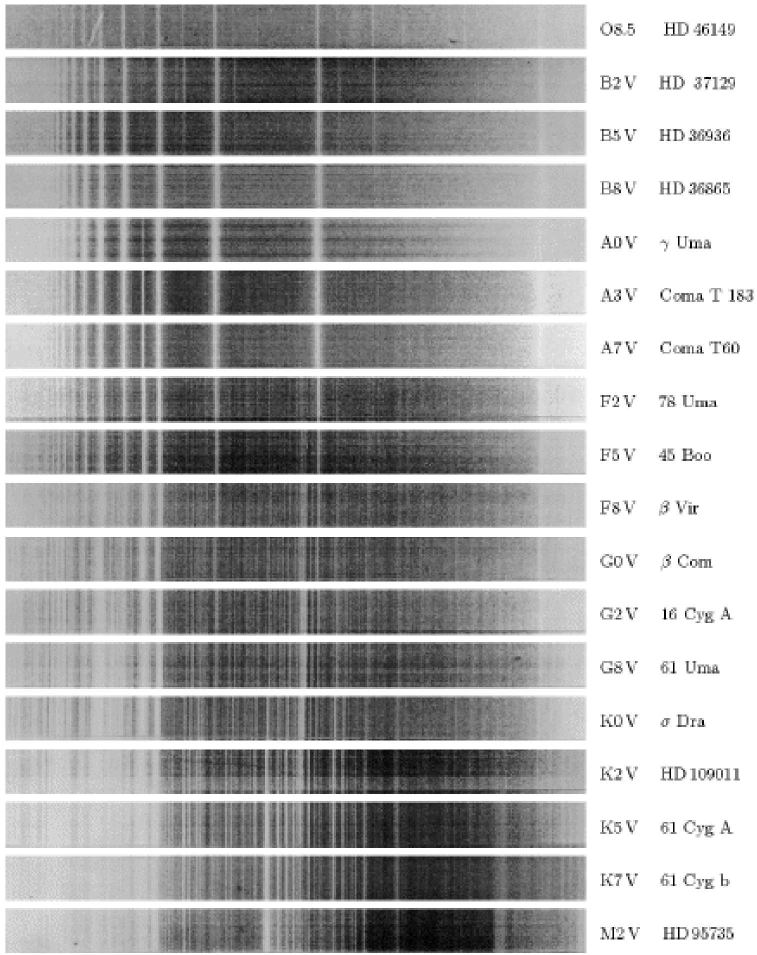
Wkipedia: *In the 1880s, the astronomer Edward C. Pickering began to make a survey of stellar spectra at the Harvard College Observatory, using the objective-prism method. A first result of this work was the Draper Catalogue of Stellar Spectra, published in 1890. Williamina Fleming classified most of the spectra in this catalogue. It used a scheme in which the previously used Secchi classes (I to IV) were divided into more specific classes, given letters from A to N. Also, the letters O, P and Q were used, O for stars whose spectra consisted mainly of bright lines, P for planetary nebulae, and Q for stars not fitting into any other class.*

In 1897, another worker at Harvard, Antonia Maury, placed the Orion subtype of Secchi class I ahead of the remainder of Secchi class I, thus placing the modern type B ahead of the modern type A. She was the first to do so, although she did not use lettered spectral types, but rather a series of twenty-two types numbered from I to XXII.

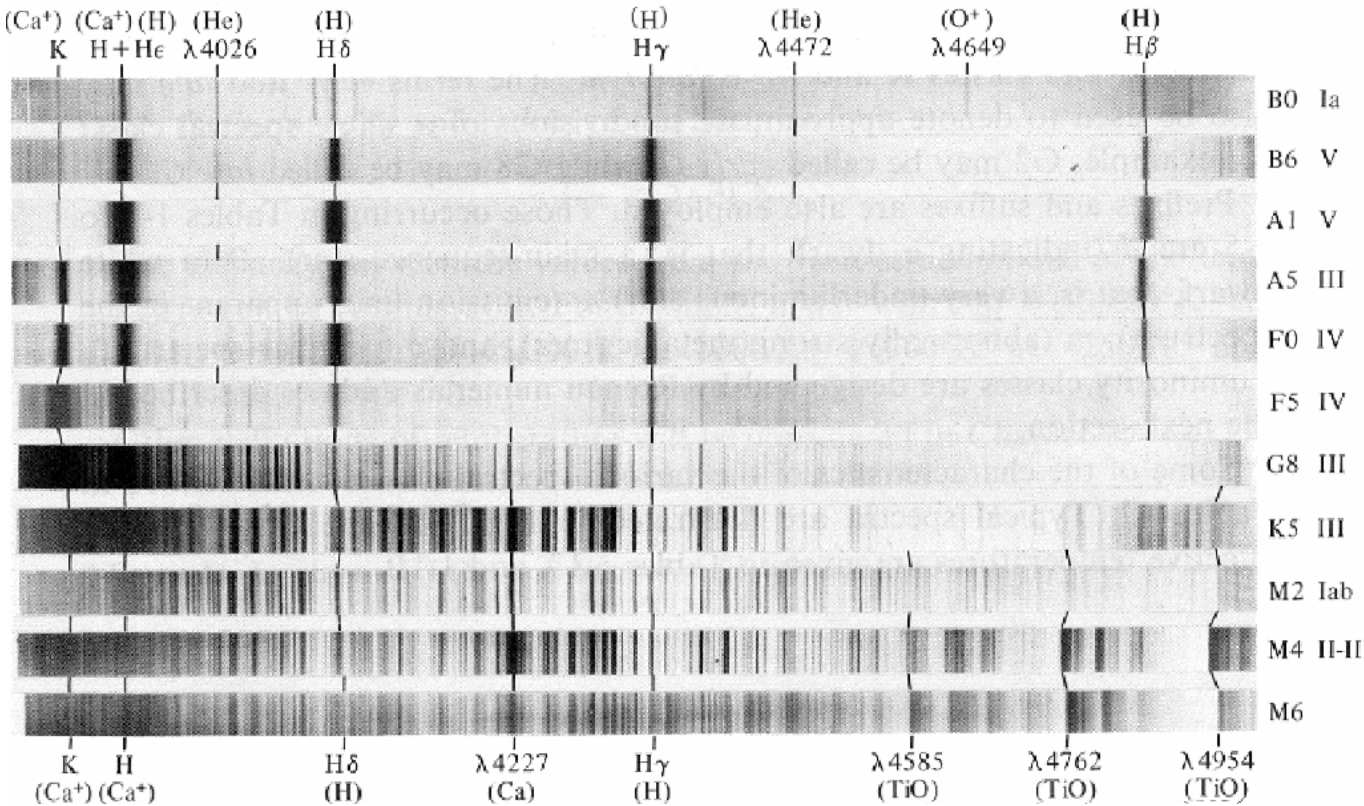
In 1901, Annie Jump Cannon returned to the lettered types, but dropped all letters except O, B, A, F, G, K, and M, used in that order, as well as P for planetary nebulae and Q for some peculiar spectra. She also used types such as B5A for stars halfway between types B and A, F2G for stars one-fifth of the way from F to G, and so forth. Finally, by 1912, Cannon had changed the types B, A, B5A, F2G, etc. to B0, A0, B5, F2, etc. This is essentially the modern form of the Harvard classification system. A common mnemonic for remembering the spectral type letters is "Oh, Be A Fine Guy/Girl, Kiss Me".

Annie Cannon

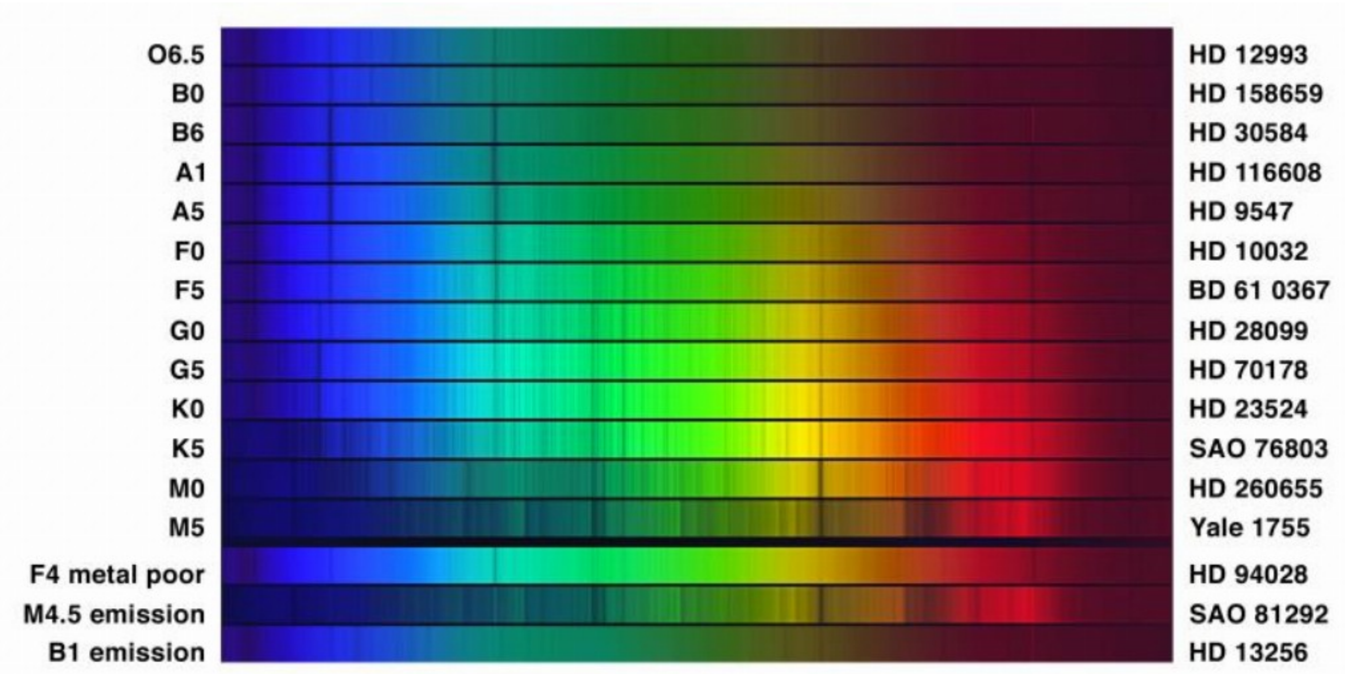
Main-sequence stellar spectra ordered



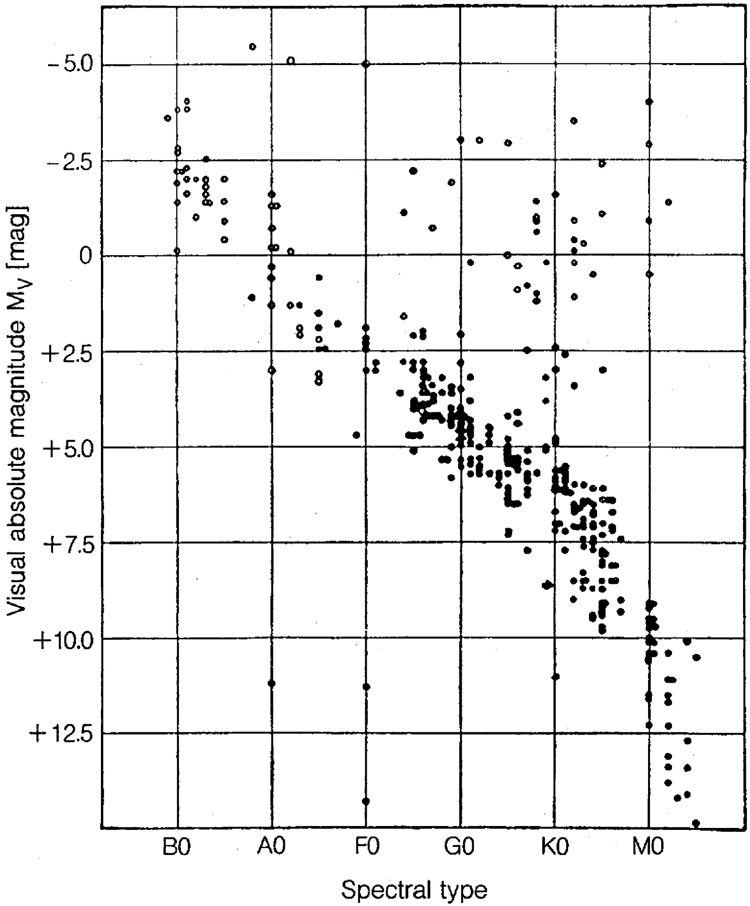
Harvard classification



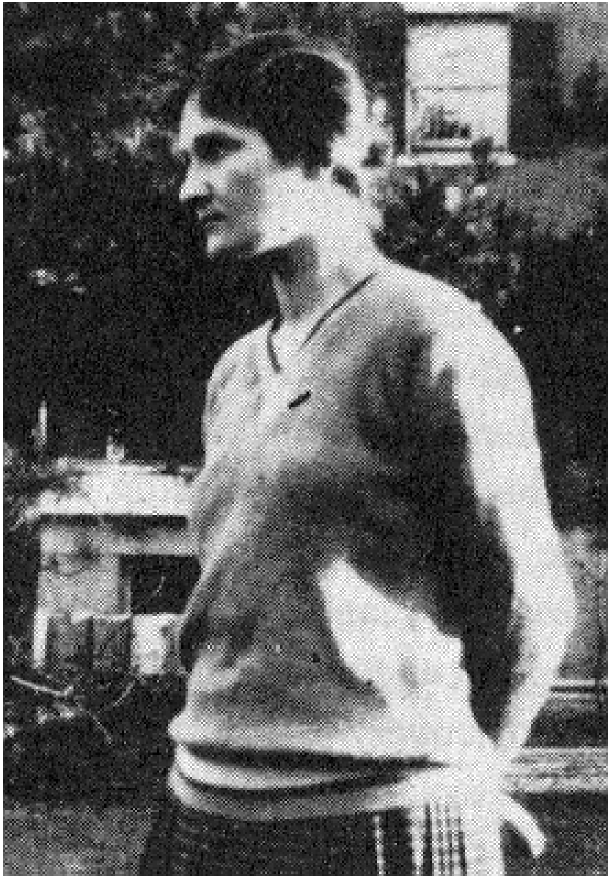
Harvard classification



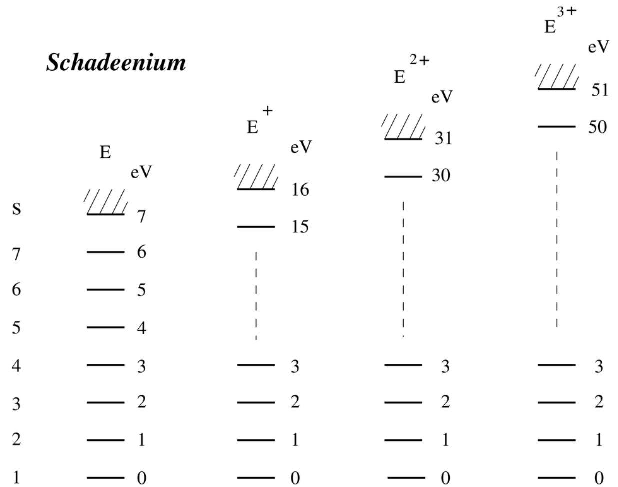
Empirical Hertzsprung-Russell diagram



Cecilia Payne



Saha-Boltzmann equations



Boltzmann distribution per ionization stage: $\frac{n_{r,s}}{N_r} = \frac{g_{r,s}}{U_r}\,\mathrm{e}^{-\chi_{r,s}/kT}$

partition function: $U_r \equiv \sum_s g_{r,s}\,\mathrm{e}^{-\chi_{r,s}/kT}$

Saha distribution over ionization stages:

$$\frac{N_{r+1}}{N_r} = \frac{1}{N_\mathrm{e}} \frac{2\,U_{r+1}}{U_r} \left(\frac{2\pi m_\mathrm{e} kT}{h^2}\right)^{3/2} \mathrm{e}^{-\chi_r/kT}$$

Schadeenium Saha-Boltzmann populations

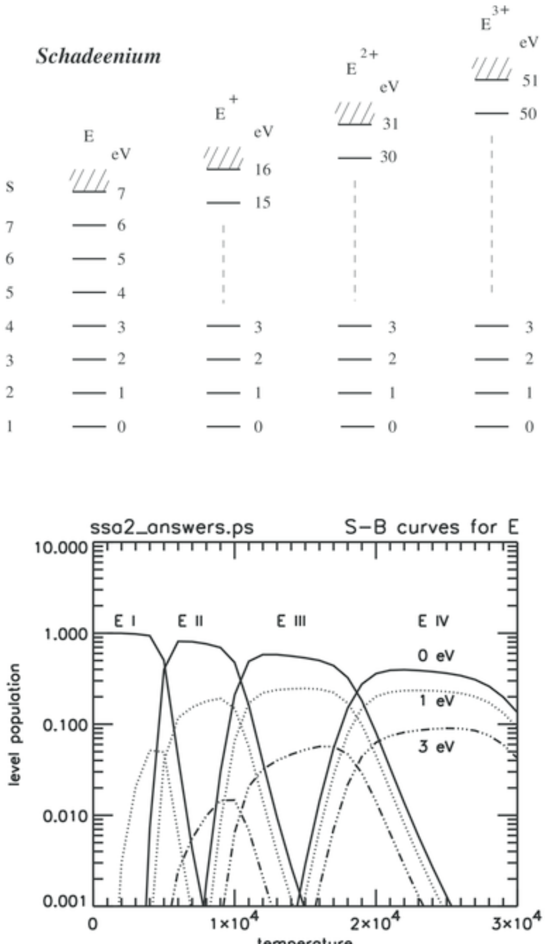
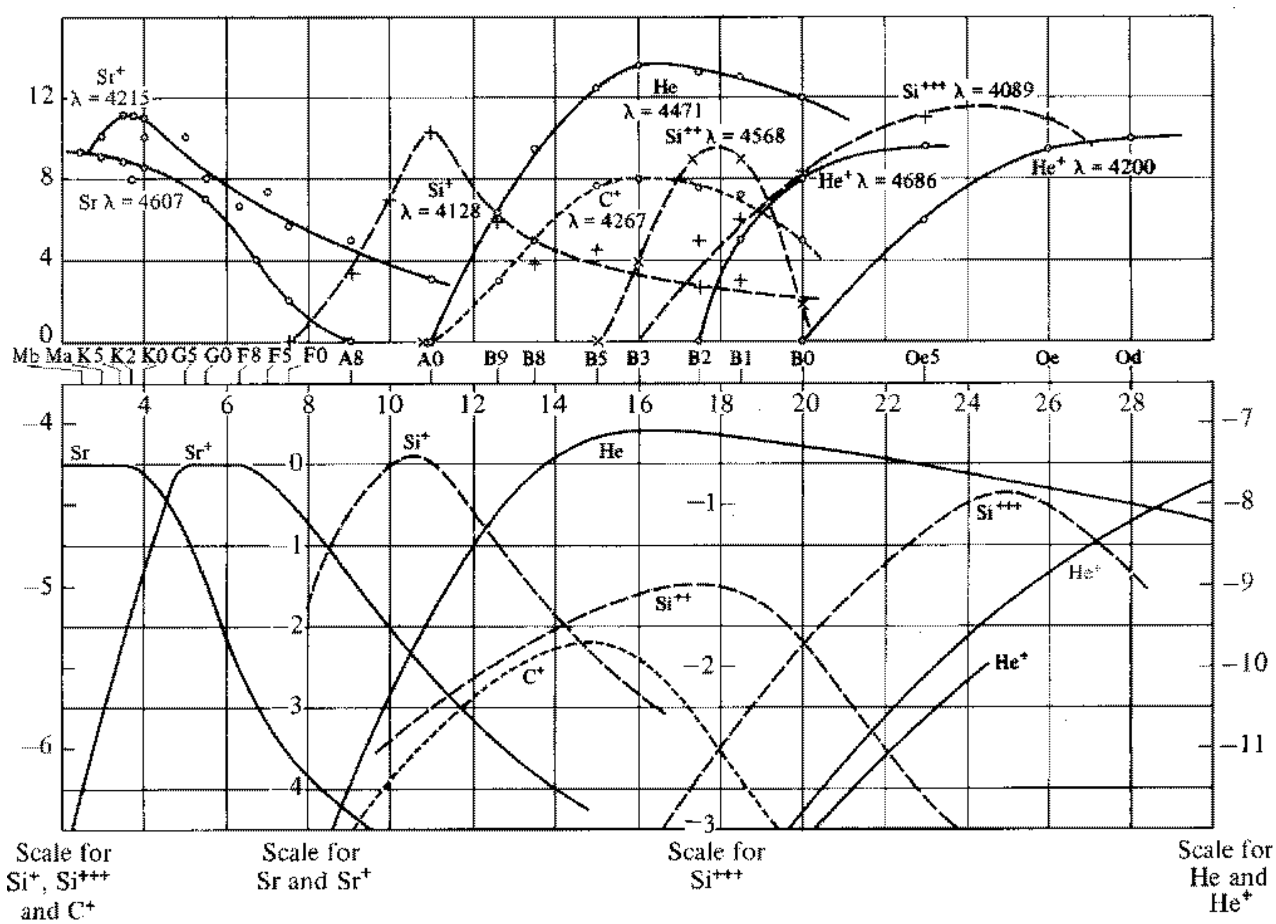
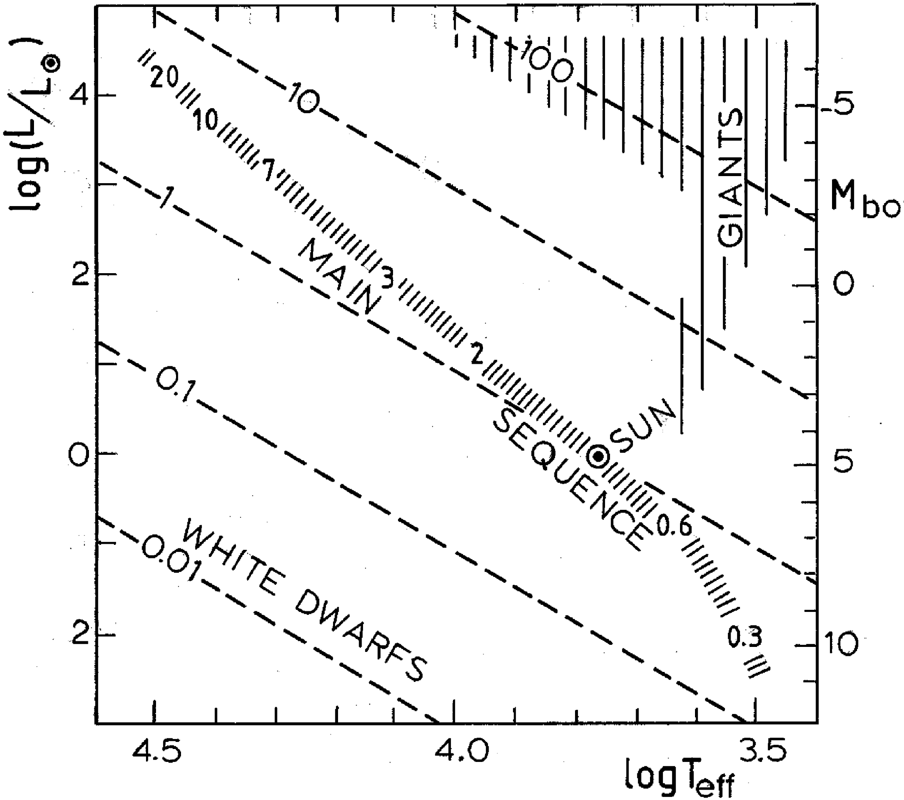


Figure 2.8: Saha-Boltzmann distributions for schadeenium, a didactic element not unlike iron invented by Aert Schadee, with symbol E. *Upper diagram*: Energy level diagram. The level energies increase in 1 eV steps. The columns may be thought stacked on top of each other since each ion requires the previous stage to be ionized. In astronomical convention the spectra of neutral schadeenium E and once-ionized schadeenium E^+ are called E I, E II, etc. *Lower diagram*: Saha-Boltzmann population fractions for levels 1, 2 and 4 of stages E I – E IV as function of temperature. All statistical weights $g_{r,s}$ were assumed unity. The population of an excited level increases with temperature until its stage ionizes. Only two stages co-exist effectively at any temperature. From my second "Stellar Spectra A" exercise at `http://www.astro.uu.nl/~rutten`. Aert Schadee (1936 – 1999) was an astrophysicist at Utrecht.

Cannon’s classification and Payne’s Saha-Boltzmann curves



Physical Hertzsprung-Russell diagram



$$L = 4\pi R^2\, \sigma T_{\rm eff}^4 \qquad (1)$$

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Exercises SSA2, SSA3

Exercises SSB

Exercises SSC

LITERATURE

- *Mihalas*
  - "Stellar Atmospheres" (1970) (?)
  - "Stellar Atmospheres" (1978) (*)
  - with Hubený: "Theory of Stellar Atmospheres" (2014)
- *simpler*
  - Novotny: "Introduction to stellar atmospheres and interiors" (1973)
  - Gray: "The observation and analysis of stellar photospheres" (2005)
  - Böhm-Vitense: "Introduction to stellar astrophysics I, II" (1989)
- *harder*
  - Cannon: "The transfer of spectral line radiation" (1985)
  - Mihalas & Mihalas: "Foundations of Radiation Hydrodynamics" (1984) (*)
  - Castor: "Radiation Hydrodynamics" (2004)
- *my stuff*
  - IART: bachelors-level radiative transfer (1991, 2015, 20??) (*)
  - RTSA: masters-level Mihalas popularization (2003, 20??) (*)
  - ISSF: bachelors-level introduction to NLTE (1993) (*)
  - Monterey: PhD-level refresher NLTE chromospheric lines for IRIS (2012) (*)
  - Cartagena: tutorial non-E hydrogen diagnostics for ALMA (2016) (*)

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BASIC QUANTITIES

Monochromatic emissivity

$$\mathrm{d}E_\nu \equiv j_\nu \,\mathrm{d}V \,\mathrm{d}t \,\mathrm{d}\nu \,\mathrm{d}\Omega \qquad \mathrm{d}I_\nu(s) = j_\nu(s)\,\mathrm{d}s$$

units j_ν: erg cm^{-3} s^{-1} Hz^{-1} ster^{-1} I_ν: erg cm^{-2} s^{-1} Hz^{-1} ster^{-1}

Monochromatic extinction coefficient

$$\mathrm{d}I_\nu \equiv -\sigma_\nu n\, I_\nu\,\mathrm{d}s \qquad \mathrm{d}I_\nu \equiv -\alpha_\nu\, I_\nu\,\mathrm{d}s \qquad \mathrm{d}I_\nu \equiv -\kappa_\nu \rho\, I_\nu\,\mathrm{d}s$$

units: cm^2 per particle (physics) cm^2 per cm^3 = per cm (RTSA) cm^2 per gram (astronomy)

Monochromatic source function

$$S_\nu \equiv j_\nu/\alpha_\nu = j_\nu/\kappa_\nu\rho \qquad S_\nu^{\mathrm{tot}} = \frac{\sum j_\nu}{\sum \alpha_\nu} \qquad S_\nu^{\mathrm{tot}} = \frac{j_\nu^c + j_\nu^l}{\alpha_\nu^c + \alpha_\nu^l} = \frac{S_\nu^c + \eta_\nu S_\nu^l}{1+\eta_\nu} \qquad \eta_\nu \equiv \alpha_\nu^l/\alpha_\nu^c$$

thick: (α_ν, S_ν) more independent than (α_ν, j_ν) stimulated emission negatively into α_ν, κ_ν

Transport equation with τ_ν as optical thickness along the beam

$$\frac{\mathrm{d}I_\nu}{\mathrm{d}s} = j_\nu - \alpha_\nu I_\nu \qquad \frac{\mathrm{d}I_\nu}{\alpha_\nu\,\mathrm{d}s} = S_\nu - I_\nu \qquad \mathrm{d}\tau_\nu \equiv \alpha_\nu \mathrm{d}s \qquad \tau_\nu(D) = \int_0^D \alpha_\nu \mathrm{d}s \qquad \frac{\mathrm{d}I_\nu}{\mathrm{d}\tau_\nu} = S_\nu - I_\nu$$

Plane-parallel transport equation with τ_ν as radial optical depth and μ as viewing angle

$$\mathrm{d}\tau_\nu \equiv -\alpha_\nu\,\mathrm{d}z \qquad \tau_\nu(z_0) = -\int_\infty^{z_0} \alpha_\nu\,\mathrm{d}z \qquad \mu \equiv \cos\theta \qquad \mu\,\frac{\mathrm{d}I_\nu}{\mathrm{d}\tau_\nu} = I_\nu - S_\nu$$

FLUX

Monochromatic flux [erg s^{-1} cm^{-2} Hz^{-1}] or [W m^{-2} Hz^{-1}] (solid angle)

$$\mathcal{F}_\nu(\vec{r},\vec{n},t) \equiv \int I_\nu \cos\theta \,\mathrm{d}\Omega = \int_0^{2\pi}\int_0^{\pi} I_\nu \cos\theta \sin\theta \,\mathrm{d}\theta\,\mathrm{d}\varphi$$

Ingoing and outgoing

$$\mathcal{F}_\nu(z) = \int_0^{2\pi}\int_0^{\pi/2} I_\nu \cos\theta\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\varphi + \int_0^{2\pi}\int_{\pi/2}^{\pi} I_\nu \cos\theta\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\varphi \equiv \mathcal{F}_\nu^+(z) - \mathcal{F}_\nu^-(z)$$

Axial symmetry (plane-parallel layers)

$$\mathcal{F}_\nu(z) = 2\pi\int_0^{\pi} I_\nu\cos\theta\sin\theta\,\mathrm{d}\theta = 2\pi\int_0^1 \mu I_\nu\,\mathrm{d}\mu \;-\; 2\pi\int_0^{-1}\mu I_\nu\,\mathrm{d}\mu = \mathcal{F}_\nu^+(z) \;-\; \mathcal{F}_\nu^-(z)$$

Surface flux of a non-irradiated spherical star

$\mathcal{F}_\nu^{\mathrm{surface}} \equiv \mathcal{F}_\nu^+(r=R) = \pi\overline{I_\nu^+}$ $\quad$ $\overline{I_\nu^+}$ = average over apparent stellar disk from faraway

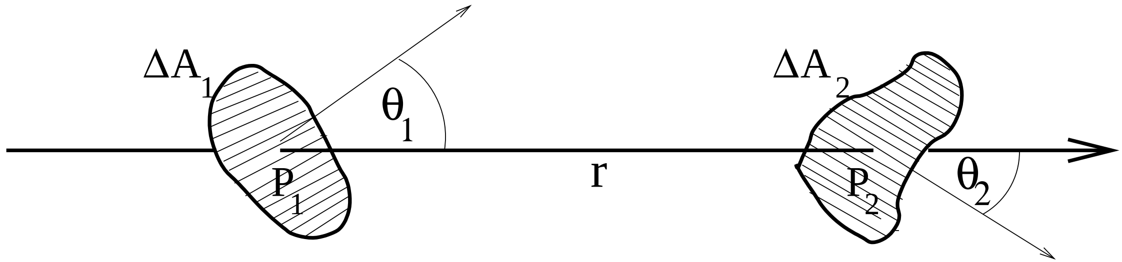
"Astrophysical flux"

$$\pi F_\nu \equiv \mathcal{F}_\nu \quad \text{so that} \quad F_\nu = \overline{I_\nu^+}$$

"Eddington flux"

$$H_\nu(z) \equiv \frac{1}{2}\int_{-1}^{+1}\mu\, I_\nu\,\mathrm{d}\mu$$

CONSERVATION OF INTENSITY



- consider all photons that travel first through area A_1 around P_1 and then also through area A_2 around P_2

- in P_1: $\Delta E_1 \equiv I_1 \, \cos\theta_1 \, \Delta A_1 \, \Delta t \, \Delta\nu \, \Delta\Omega_1$.
  This proportionality holds in the infinitesimal limit $\Delta \rightarrow \mathrm{d}$

- likewise in P_2: $\Delta E_2 \equiv I_2 \, \cos\theta_2 \, \Delta A_2 \, \Delta t \, \Delta\nu \, \Delta\Omega_2$

- insert the solid angle that each area extends on the sky of the other:
  $\Delta\Omega_1 = \cos\theta_2 \, \Delta A_2 / r^2$ and $\Delta\Omega_2 = \cos\theta_1 \, \Delta A_1 / r^2$
  $\Delta E_1 \equiv I_1 \, \cos\theta_1 \, \Delta A_1 \, \Delta t \, \Delta\nu \, \cos\theta_2 \, \Delta A_2 \, / \, r^2 = I_1 \, \cos\theta_1 \, \cos\theta_2 \, \Delta A_1 \, \Delta A_2 \, \Delta t \, \Delta\nu \, / \, r^2$
  $\Delta E_2 \equiv I_2 \, \cos\theta_2 \, \Delta A_2 \, \Delta t \, \Delta\nu \, \cos\theta_1 \, \Delta A_1 / \, r^2 = I_2 \, \cos\theta_1 \, \cos\theta_2 \, \Delta A_1 \, \Delta A_2 \, \Delta t \, \Delta\nu \, / \, r^2$

- since $\Delta E_1 = \Delta E_2$ (the given stream of photons) it follows that $I_1 = I_2$

This macroscopic conservancy for the propagation of light in vacuum expresses the photon property of non-decay (mass zero).

- You can use a magnifying glass to start a fire with sunshine. What is the intensity in its focus? Why does it heat?

- The 4-m DKIST will have four times the aperture size of the 1-m SST. Compare the exposure times needed for solar observation.

  Christoph Keller's answer

- A supergiant telescope resolves granules on a solar-analog star 10 lightyears away. What exposure is needed compared to DKIST?

- An amateur astronomer in Iceland photographs an Apollo landing site on the Moon through her 25-cm telescope at 100 times magnification with a Canon camera. Compare the required exposure time to when she uses her Canon with its standard lens in the Holuhraun.

  My answer

- Why are the largest solar telescopes smaller than the largest night-time telescopes?

CONSTANT SOURCE FUNCTION

Transport equation along the beam (τ_ν = optical thickness)

$$\frac{\mathrm{d}I_\nu}{\mathrm{d}\tau_\nu} = S_\nu - I_\nu \qquad\qquad I_\nu(\tau_\nu) = I_\nu(0)\,\mathrm{e}^{-\tau_\nu} + \int_0^{\tau_\nu} S_\nu(t_\nu)\,\mathrm{e}^{-(\tau_\nu - t_\nu)}\,\mathrm{d}t_\nu$$

Invariant S_ν

$$I_\nu(D) = I_\nu(0)\,\mathrm{e}^{-\tau_\nu(D)} + S_\nu\left(1 - \mathrm{e}^{-\tau_\nu(D)}\right)$$

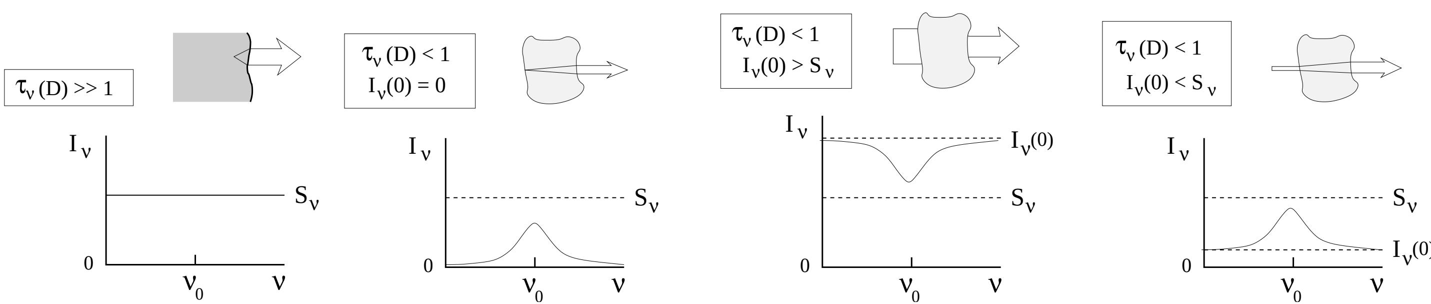
example: $S_\nu = B_\nu$ for all continuum and line processes in an isothermal cloud

Thick object

$$I_\nu(D) \approx S_\nu$$

Thin object

$$I_\nu(D) \approx I_\nu(0) + [S_\nu - I_\nu(0)]\;\tau_\nu(D) \qquad\qquad I_\nu(0)\!=\!0:\;\; I_\nu(D) \approx \tau_\nu(D)\,S_\nu = j_\nu D$$



RADIATIVE TRANSFER IN A PLANE ATMOSPHERE

Radial optical depth

$$\mathrm{d}\tau_\nu = -\kappa_\nu \rho \, \mathrm{d}r$$

r radial   Hubený $\tau_{\nu\mu}$ κ_ν cm^2/gram α_ν cm^{-1} = cm^2/cm^3 σ_ν cm^2/particle

Transport equation

$$\mu \frac{\mathrm{d}I_\nu}{\mathrm{d}\tau_\nu} = I_\nu - S_\nu$$

Integral form

$$I_\nu^-(\tau_\nu, \mu) = -\int_0^{\tau_\nu} S_\nu(t_\nu) \, \mathrm{e}^{-(t_\nu - \tau_\nu)/\mu} \, \mathrm{d}t_\nu/\mu \qquad I_\nu^+(\tau_\nu, \mu) = +\int_{\tau_\nu}^{\infty} S_\nu(t_\nu) \, \mathrm{e}^{-(t_\nu - \tau_\nu)/\mu} \, \mathrm{d}t_\nu/\mu$$

"formal solution"   NB: both directions   pm: Doppler anisotropy S_ν

Emergent intensity without irradiation

$$I_\nu(0, \mu) = (1/\mu) \int_0^{\infty} S_\nu(\tau_\nu) \; \mathrm{e}^{-\tau_\nu/\mu} \, \mathrm{d}\tau_\nu$$

Eddington–Barbier approximation

$$\boxed{I_\nu(0, \mu) \approx S_\nu(\tau_\nu \!=\! \mu)}$$

exact for linear $S_\nu(\tau_\nu)$

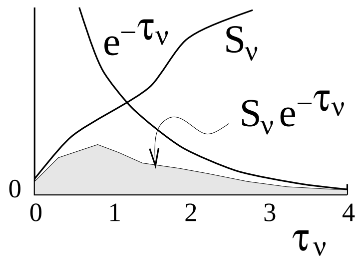
EDDINGTON-BARBIER APPROXIMATION

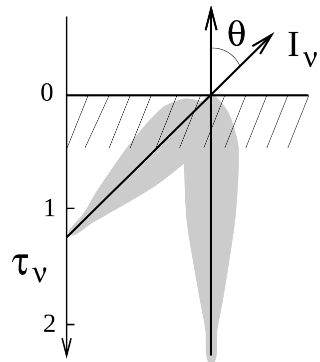
Emergent intensity without irradiation

$$I_\nu(0,\mu) = (1/\mu) \int_0^\infty S_\nu(\tau_\nu)\ \mathrm{e}^{-\tau_\nu/\mu}\ \mathrm{d}\tau_\nu$$

Eddington-Barbier (Milne–Unsöld?) approximation

$$\boxed{I_\nu(0,\mu) \approx S_\nu(\tau_\nu\!=\!\mu)}$$





- wrong: *"the radiation comes from $\tau_\nu\!=\!1$"* or *"the photons escape at $\tau_\nu\!=\!1$"*
- correct: *"the emergent radiation is characterized by the source function at $\tau_\nu\!=\!1$"*
- beware: a spectral line may instead be formed in a "cloud" at any height
- unresolved star: $F_\nu(0) \approx S(\tau_\nu\!=\!2/3)$ with $F_\nu(0) = 2\int_0^1 \mu I_\nu(0)\,\mathrm{d}\mu = \overline{I_\nu(0)}$

CONTRIBUTION & RESPONSE FUNCTIONS

Eddington-Barbier approximation

$$I_\nu(0,\mu) \approx S_\nu(\tau_\nu\!=\!\mu)$$

Intensity contribution function

$$I_\nu = \int S_\nu \;\mathrm{e}^{-\tau_\nu}\,\mathrm{d}\tau_\nu \qquad \mathrm{CF}_I \equiv \frac{\mathrm{d}I_\nu}{\mathrm{d}h} = S_\nu\;\mathrm{e}^{-\tau_\nu}\,\frac{\mathrm{d}\tau_\nu}{\mathrm{d}h} = j_\nu\;\mathrm{e}^{-\tau_\nu}$$

Line depression contribution function (Magain 1986A&A...163..135M)

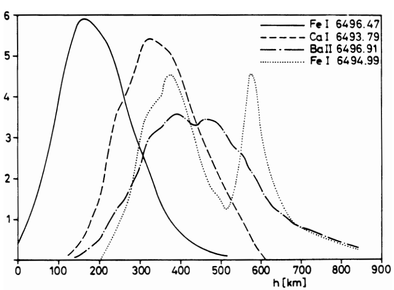
$$D \equiv \frac{I_\nu^C - I_\nu}{I_\nu^C} = \int S_D\,\mathrm{e}^{-\tau_D}\,\mathrm{d}\tau_D \qquad S_D = \frac{1 - S_l/I_C}{1 + (\kappa_C/\kappa_l)\,S_C/I_C} \qquad \kappa_D = \kappa_l + \kappa_C(S_C/I_C)$$

Intensity response function

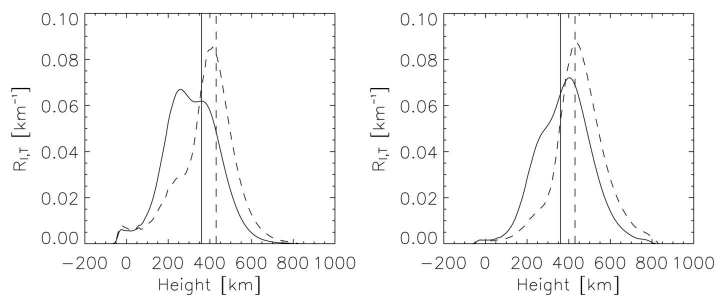
$$I_\nu = \int_{-\infty}^{+\infty} R_{I,X}(\nu,h)\,X(h)\,\mathrm{d}h \qquad \Delta I_\nu = \int_{-\infty}^{+\infty} R_{I,X}(\nu,h)\,\Delta X(h)\,\mathrm{d}h$$

Numerical intensity response function (Fossum & Carlsson 2005ApJ...625..556F)

$$\Delta X(h') = x(h')\,H(h'\!-\!h) \qquad \Delta I_\nu^h = \int_{-\infty}^{h} R_{I,X}(\nu,h')\,x(h')\,\mathrm{d}h' \qquad R_{I,X}(\nu,h) = \frac{1}{x(h)}\,\frac{\mathrm{d}\Delta I_\nu^h}{\mathrm{d}h}$$



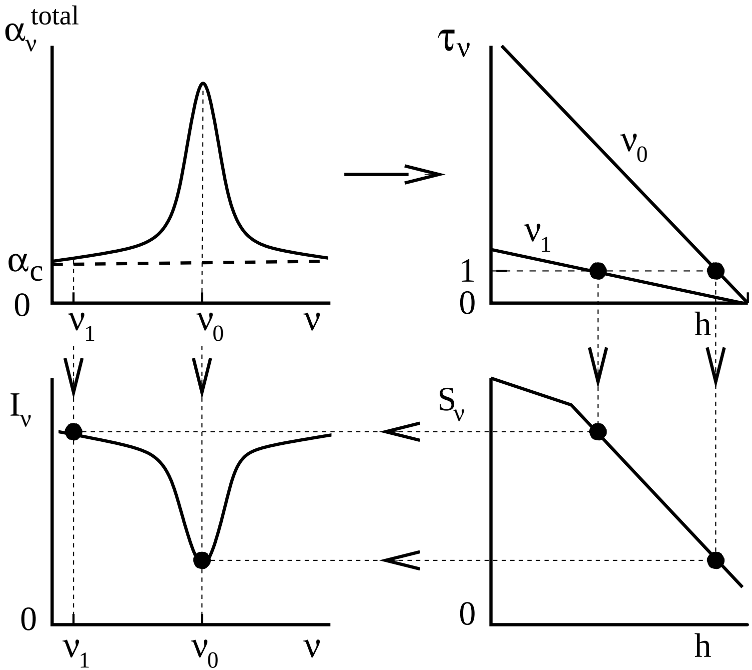
Stix Fig. 6.9



Fossum & Carlsson 2005ApJ...625..556F

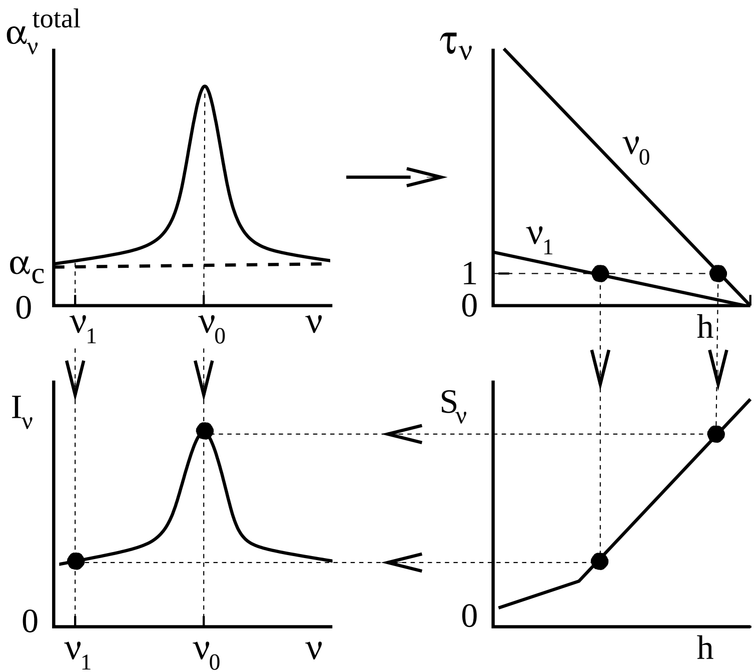
SIMPLE ABSORPTION LINE

- extinction: bb process gives peak in $\alpha_{\rm total} = \alpha_c + \alpha_l = (1+\eta_\nu)\,\alpha_c$
- optical depth: assume height-invariant $\alpha_{\rm total} \Rightarrow$ linear $(1+\eta_\nu)\,\tau_c$
- source function: assume same for line (bb) and continuous (bf, ff) processes
- use Eddington-Barbier (here nearly exact, why?)



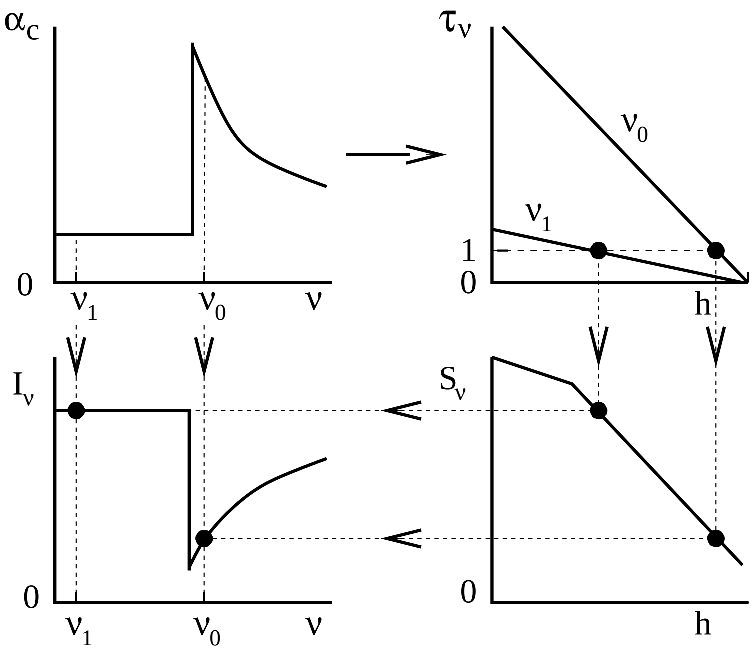
SIMPLE EMISSION LINE

- extinction: bb process gives peak in $\alpha_{\rm total} = \alpha_c + \alpha_l = (1 + \eta_\nu)\,\alpha_c$
- optical depth: assume height-invariant $\alpha_{\rm total} \Rightarrow$ linear $(1 + \eta_\nu)\,\tau_c$
- source function: assume same for line (bb) and continuous (bf, ff) processes
- use Eddington-Barbier (here nearly exact, why?)



SIMPLE ABSORPTION EDGE

- extinction: bf process gives edge in α^c_ν, with $\alpha^c_\nu \propto \nu^3$ if hydrogenic
- optical depth: assume height-invariant (unrealistic, why?)
- source function: assume same for the whole frequency range (unrealistic, why?)
- use Eddington-Barbier (here nearly exact, why?)



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- extinction: bf process gives edge in α_ν^c, with $\alpha_\nu^c \propto \nu^3$ if hydrogenic
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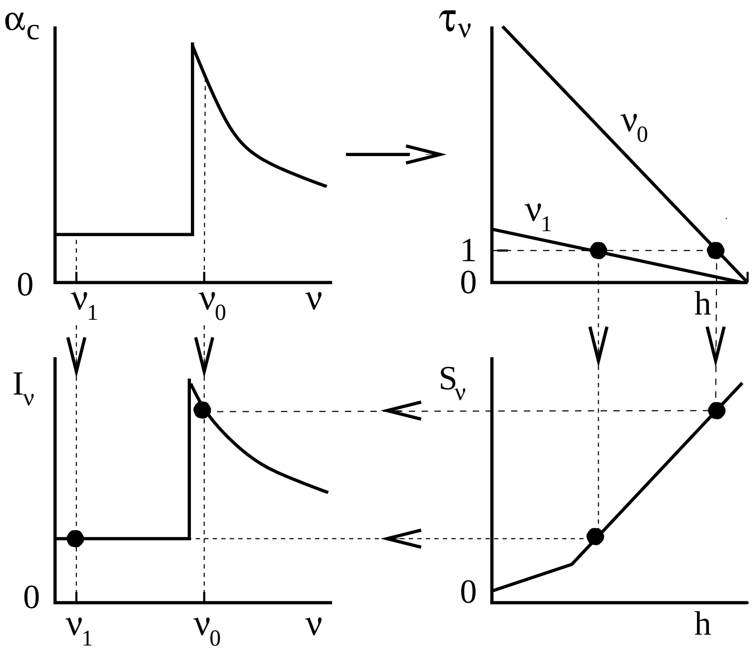
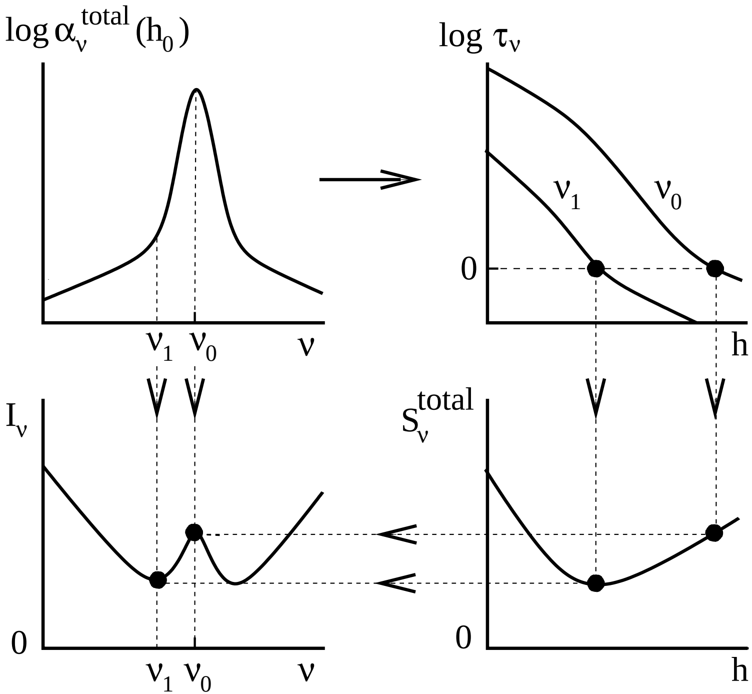


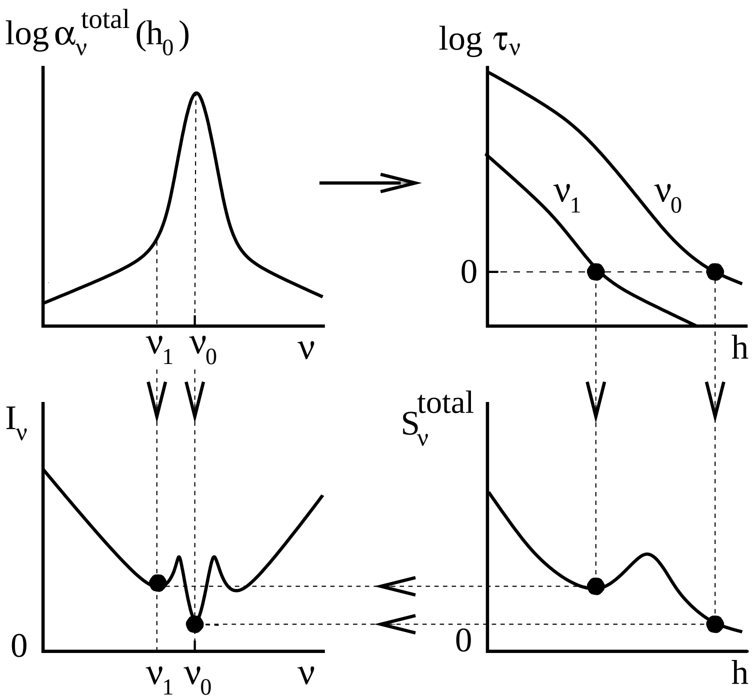
SELF-REVERSED ABSORPTION LINE

- extinction: bb peak with height-dependent amplitude and shape
- optical depth: non-linear even in the log
- source function: decrease followed by increase (any idea why?)
- use Eddington-Barbier (questionable, why?)



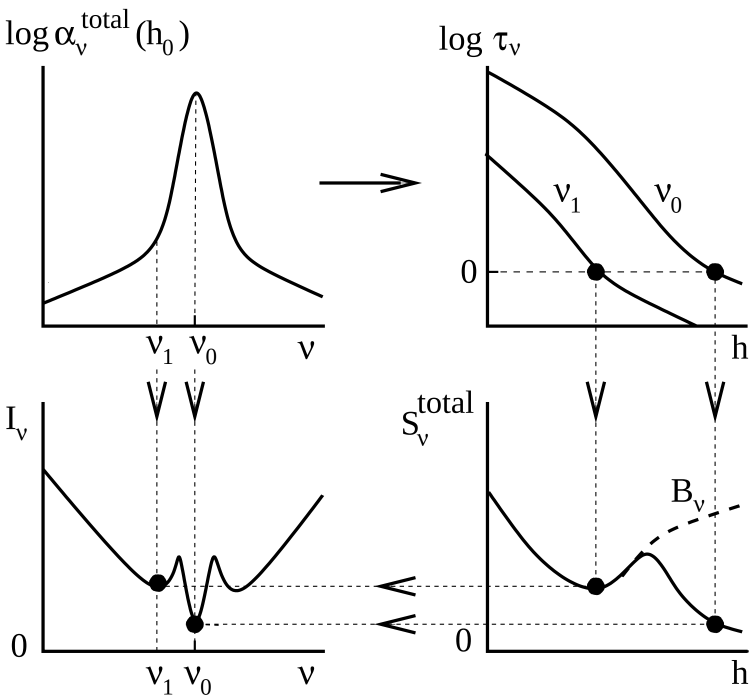
DOUBLY REVERSED ABSORPTION LINE

- extinction: bb peak with height-dependent amplitude and shape
- optical depth: non-linear even in the log
- source function: decrease followed by increase followed by decrease (any idea why?)
- use Eddington-Barbier (questionable, why?)



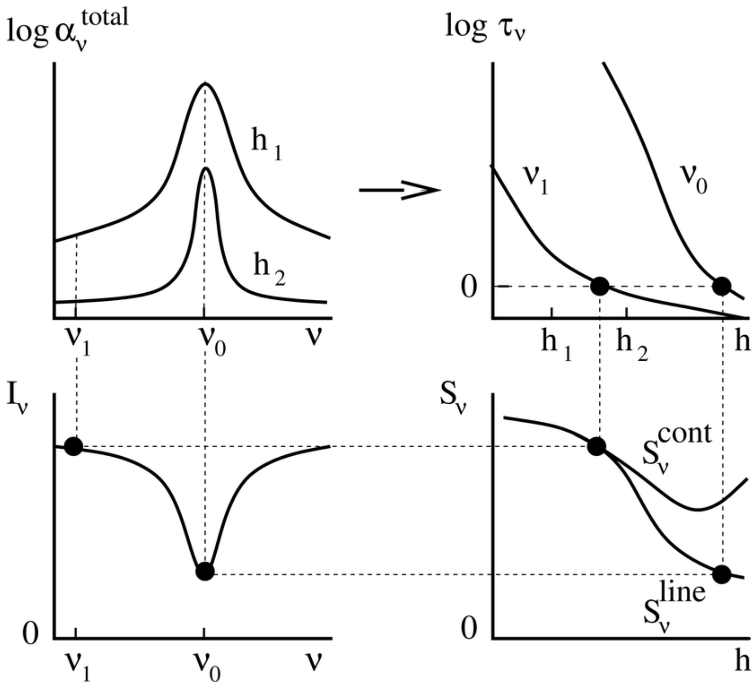
DOUBLY REVERSED ABSORPTION LINE

- extinction: bb peak with height-dependent amplitude and shape
- optical depth: non-linear even in the log
- source function: decrease followed by increase followed by decrease (NLTE scattering)
- use Eddington-Barbier (questionable, why?)



REALISTIC SOLAR ABSORPTION LINE

- extinction: bb peak in $\eta_\nu \equiv \alpha_l/\alpha_c$ becomes lower and narrower at larger height
- optical depth: $\tau_\nu \equiv -\int \alpha_\nu^{\rm total}\,{\rm d}h$ increases nearly log-linearly with geometrical depth
- source function: split for line (bb) and continuous (bf, ff, electron scattering) processes
- intensity: Eddington-Barbier for $S_\nu^{\rm total} = (\alpha_c S_c + \alpha_l S_l)/(\alpha_c + \alpha_l) = (S_C + \eta_\nu S_l)/(1+\eta_\nu)$



SOLAR SPECTRUM FORMATION: THEORY

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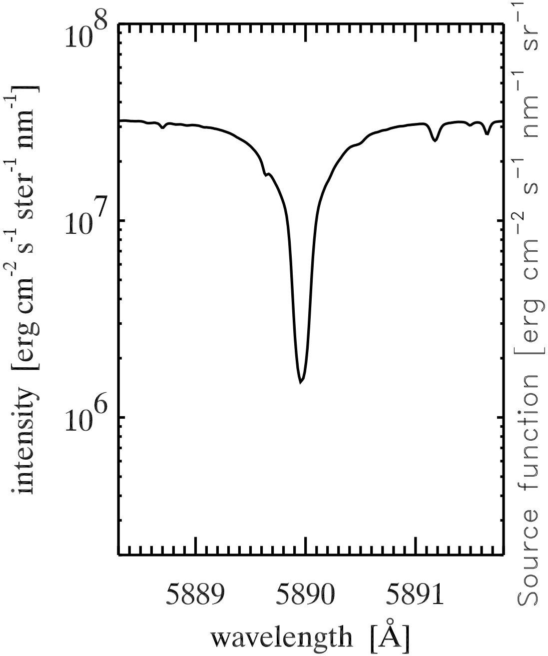
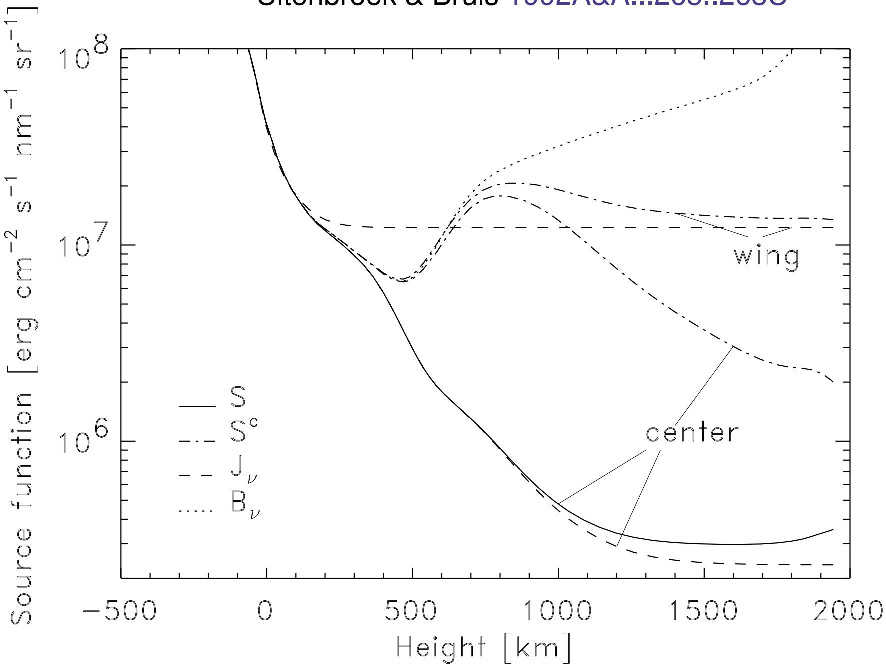
SOLAR NaI D2

NSO disk-center FTS atlas



Uitenbroek & Bruls 1992A&A...265..268U

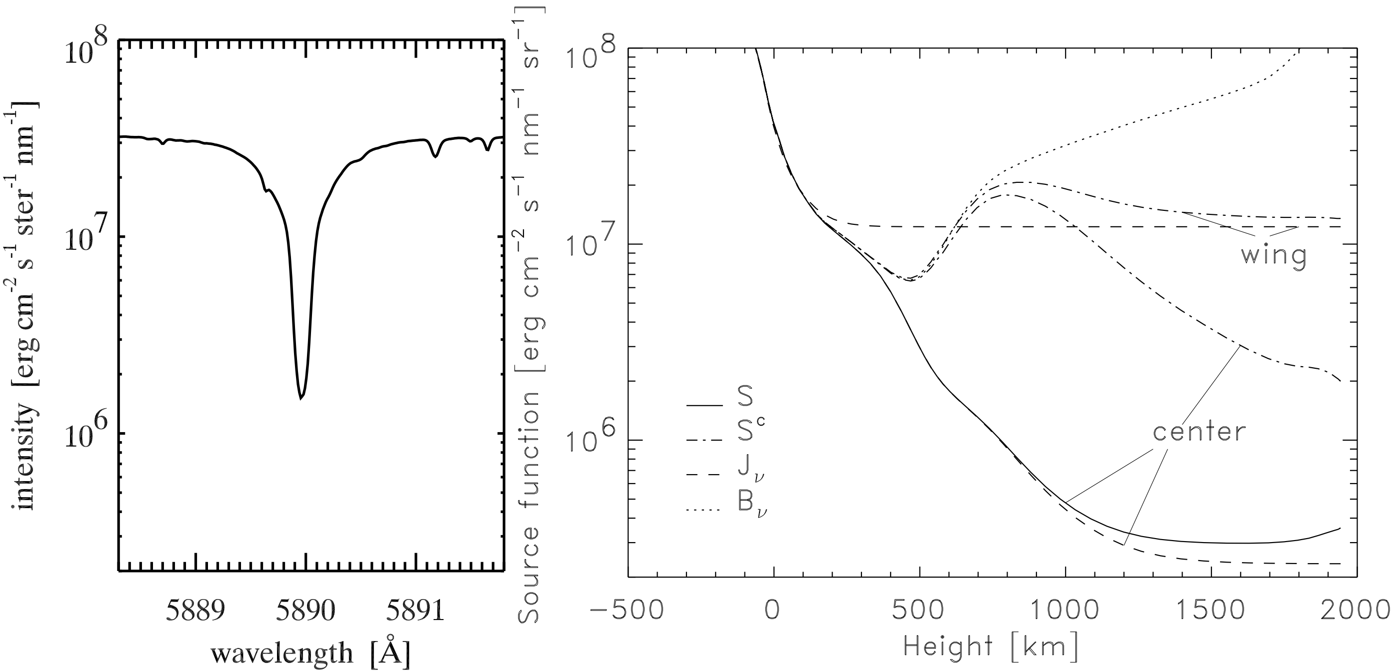


Na I D_2 is a good example of two-level scattering with complete redistribution: very dark

Eddington-Barbier approximation: line-center $\tau = 1$ at $h \approx 600$ km
chromospheric velocity response but photospheric brightness response

What is the formation height of the blend line in the blue wing?

SOLAR NaI D2



Eddington-Barbier for the blends? Moore, Minnaert, Houtgast 1966sst..book.....M:

5888.703		10.	2.	ATM H2O	R4	302	26
5889.637		14.	2.	ATM H2O	R4	401	26
5889.756	*	752.	4.	ATM H2O	R3	401	26
5889.973M	*	752.	120.SS	NA 1(D2)	0.00	1	
5890.203	*	752.	3.	ATM H2O	R4	302	26
5890.495		5.	1.S"	FE 1P	5.06	1313	
5891.178		17.	3.S	ATM H2O	R3	401	17,26
5891.178		17.	3.S	FE 1P	4.65	1179	

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Definition with $\mu \equiv 1/w$ to make them an integrals over angle

$$E_n(x) \equiv \int_1^\infty \frac{\mathrm{e}^{-xw}}{w^n}\,\mathrm{d}w = \int_0^1 \mathrm{e}^{-x/\mu}\,\mu^{n-1}\,\frac{\mathrm{d}\mu}{\mu}$$

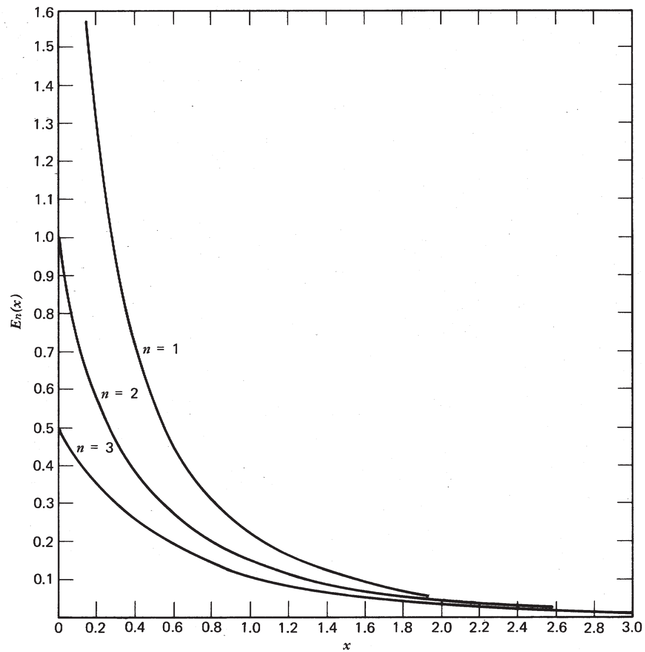


Figure 4.1: The first three exponential integrals $E_n(x)$. $E_1(x)$ has a singularity at $x = 0$. For large x all $E_n(x)$ have $E_n(x) \approx \exp(-x)/x$. From Gray (1992).

LAMBDA OPERATOR

Exponential integrals

$$E_n(x) \equiv \int_1^\infty \frac{\mathrm{e}^{-xw}}{w^n}\,\mathrm{d}w = \int_0^1 \mathrm{e}^{-x/\mu}\,\mu^{n-1}\,\frac{\mathrm{d}\mu}{\mu}$$

Schwarzschild equation

$$J_\nu(\tau_\nu) \equiv \frac{1}{2}\int_{-1}^{+1} I_\nu(\tau_\nu,\mu)\,\mathrm{d}\mu = \frac{1}{2}\int_{\tau_\nu}^\infty S_\nu(t_\nu)\,E_1(t_\nu - \tau_\nu)\,\mathrm{d}t_\nu + \frac{1}{2}\int_0^{\tau_\nu} S_\nu(t_\nu)\,E_1(\tau_\nu - t_\nu)\,\mathrm{d}t_\nu$$

$$= \frac{1}{2}\int_0^\infty S_\nu(t_\nu)\,E_1(|t_\nu - \tau_\nu|)\,\mathrm{d}t_\nu$$

Lambda operator

$$\mathbf{\Lambda}_\tau[f(t)] \equiv \frac{1}{2}\int_0^\infty f(t)\,E_1(|t-\tau|)\,\mathrm{d}t$$

Schwarzschild equation

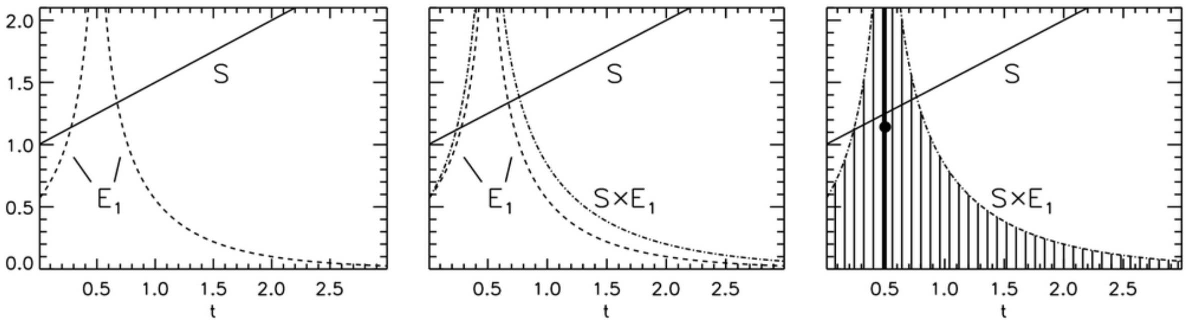
$$J_\nu(\tau_\nu) = \frac{1}{2}\int_0^\infty S_\nu(t_\nu)\,E_1(|t_\nu - \tau_\nu|)\,\mathrm{d}t_\nu = \mathbf{\Lambda}_{\tau_\nu}[S_\nu(t_\nu)]$$

Generalized lambda operators (Scharmer, Hubený)

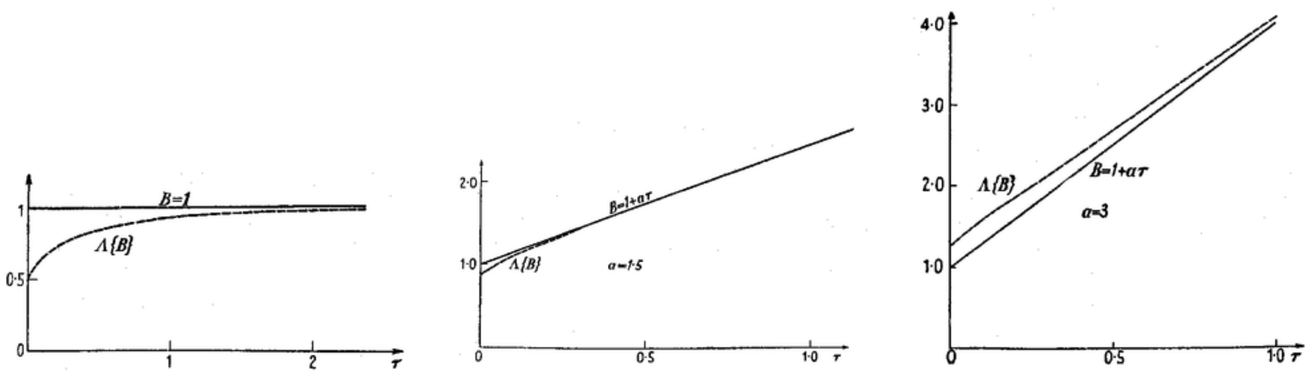
$$\overline{J}_\nu(\tau_\nu) = \overline{\mathbf{\Lambda}}\,[S_\nu(t_\nu)] \qquad\qquad I_\nu(\tau_{\nu\mu},\mu) \equiv I^\pm_{\nu\mu} = \mathbf{\Lambda}_{\nu\mu}[S_\nu(\tau_{\nu\mu})]$$

RADIATION FROM ELSEWHERE: THE Λ OPERATOR

$$J_\nu(\tau_\nu) = \mathbf{\Lambda}_{\tau_\nu}[S_\nu(t_\nu)] \equiv \frac{1}{2}\int_0^\infty S_\nu(t_\nu)\, E_1(|t_\nu - \tau_\nu|)\, \mathrm{d}t_\nu$$



Krijger 2003rtsa.book.....R



Kourganoff 1952QB801.K78......

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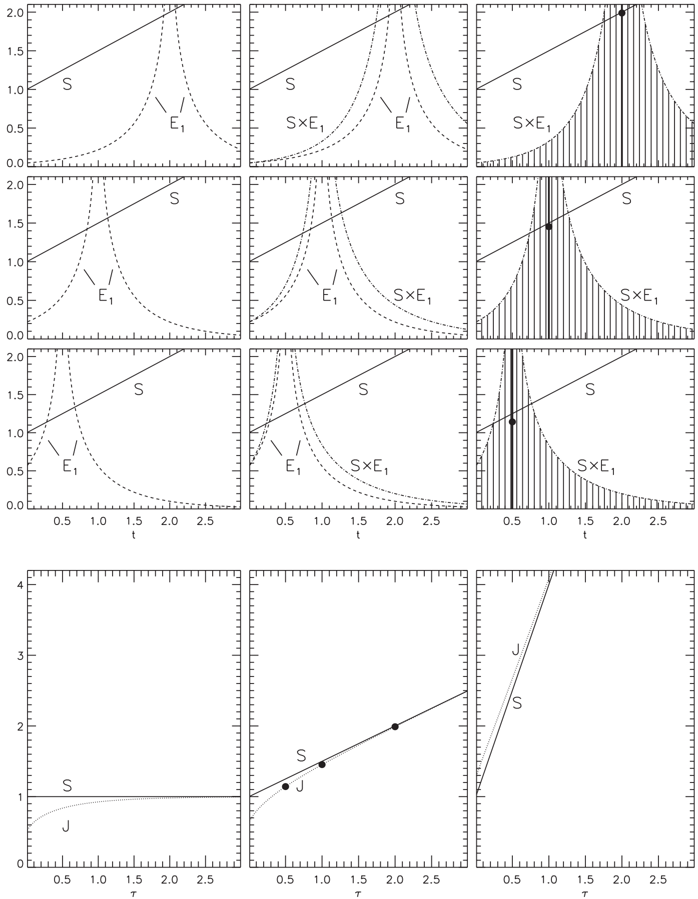
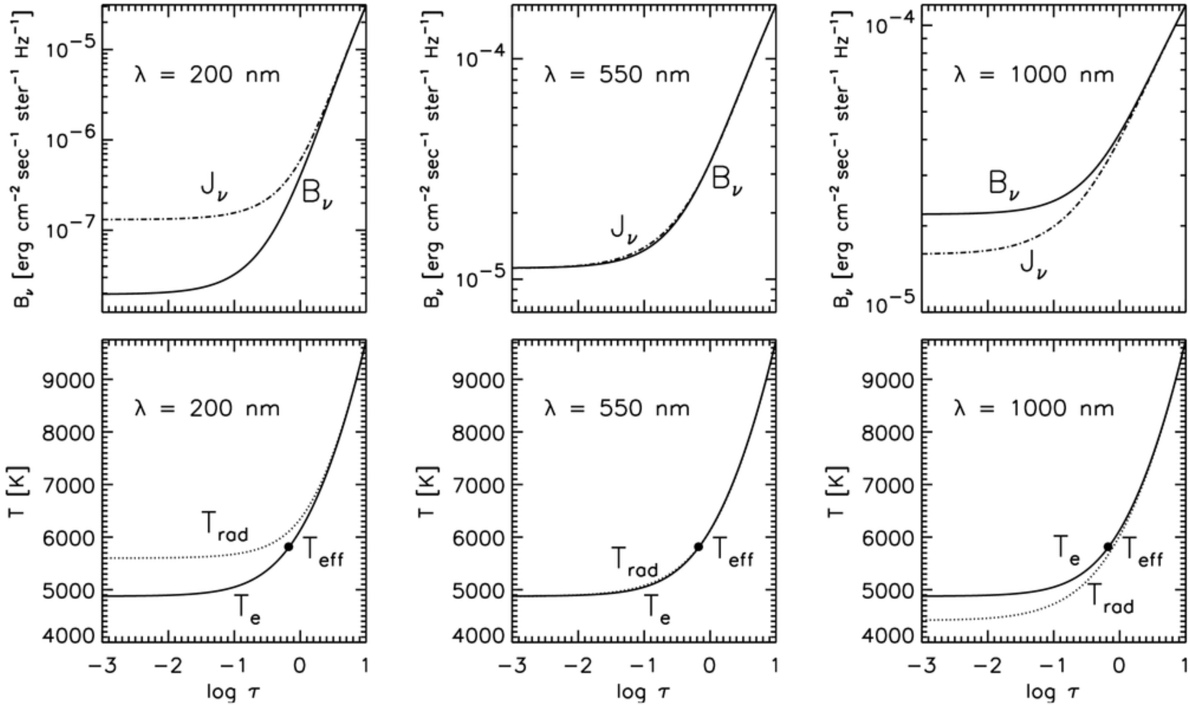
THE WORKING OF THE LAMBDA OPERATOR

RTSA figure 4.1; Thijs Krijger production



THE Λ OPERATOR FOR AN LTE-RE ATMOSPHERE

$$J_\nu(\tau_\nu) = \mathbf{\Lambda}_{\tau_\nu}[B_\nu(t_\nu)] \equiv \frac{1}{2}\int_0^\infty B_\nu(t_\nu)\,E_1(|t_\nu - \tau_\nu|)\,\mathrm{d}t_\nu$$



M.J. Krijger 1998

Conversion to formal radiation temperatures $B_\nu(T_{\rm rad}) \equiv J_\nu$ removes the wavelength dependence of the Planck function sensitivity to temperature

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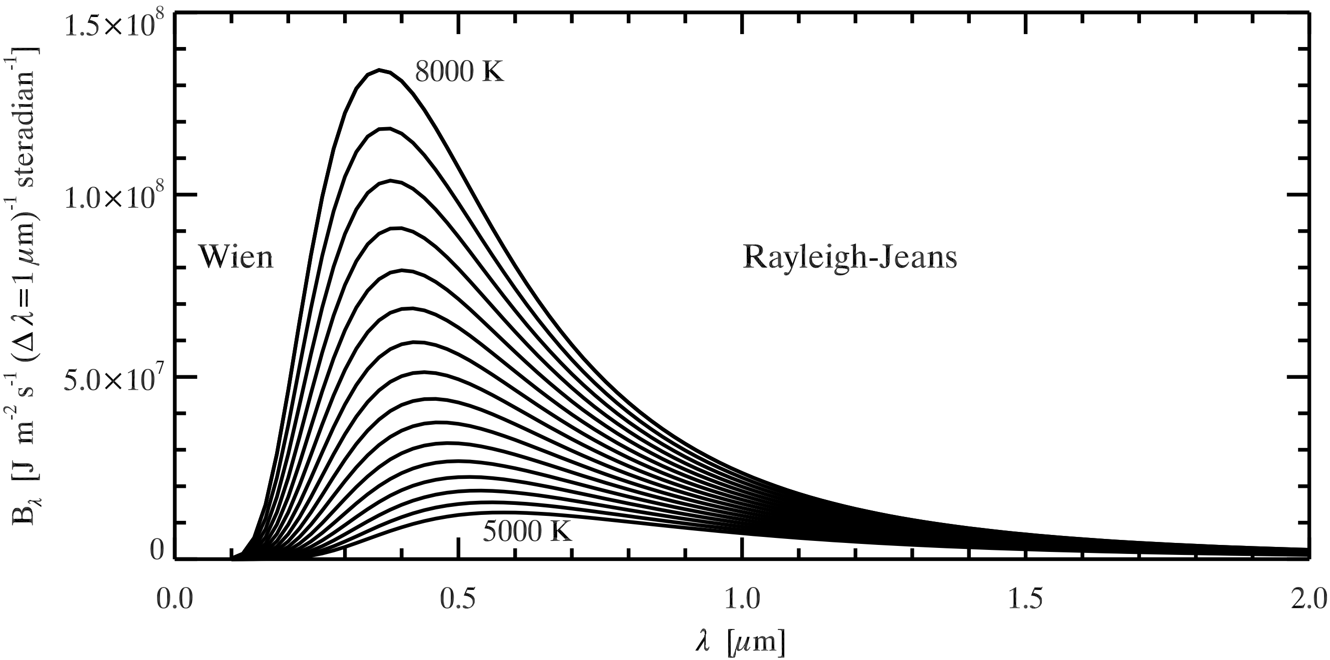
PLANCK

Planck function in intensity units

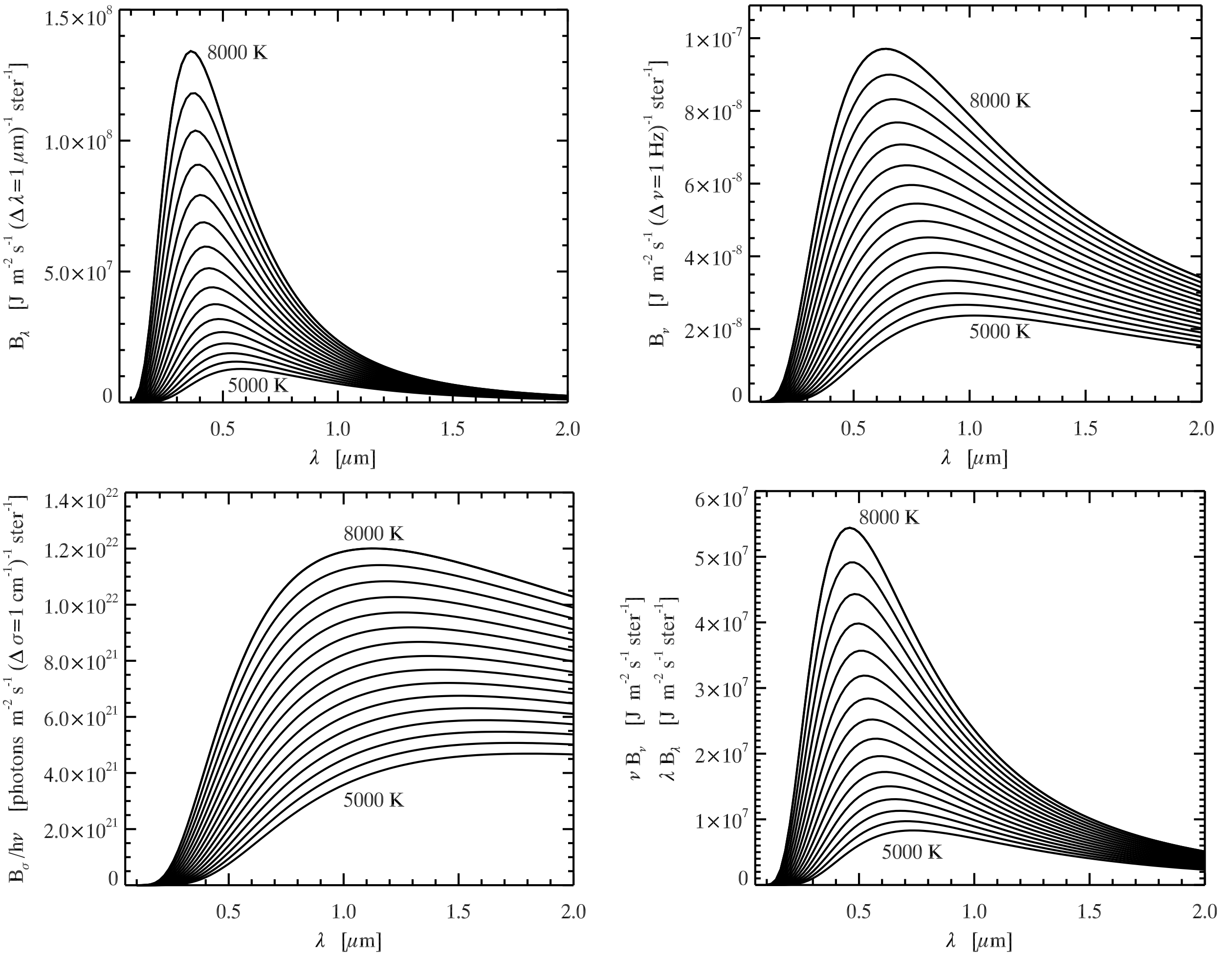
$$B_\nu(T) = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT}-1} \qquad B_\lambda(T) = \frac{2hc^2}{\lambda^5}\frac{1}{e^{hc/\lambda kT}-1} \qquad B_\sigma(T) = 2hc^2\sigma^3\frac{1}{e^{hc\sigma/kT}-1}$$

Approximations

Wien: $B_\nu(T) \approx \frac{2h\nu^3}{c^2}\, e^{-h\nu/kT}$ Rayleigh-Jeans: $B_\nu(T) \approx \frac{2\nu^2 kT}{c^2}$



PLANCK FUNCTION VARIATIONS



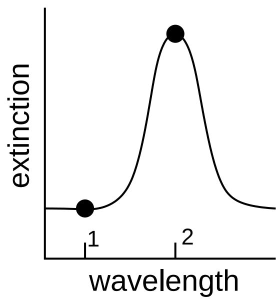
SOLAR ABSORPTION LINES AND LIMB DARKENING

Emergent intensity without irradiation

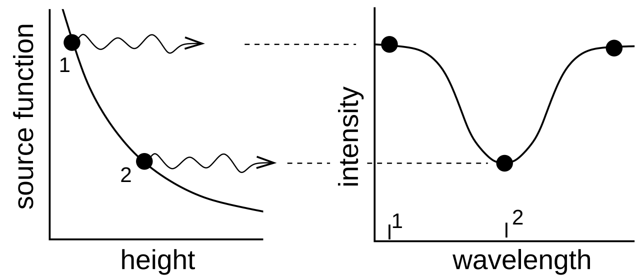
$$I_\nu(0,\mu) = (1/\mu) \int_0^\infty S_\nu(\tau_\nu)\ \mathrm{e}^{-\tau_\nu/\mu}\,\mathrm{d}\tau_\nu$$

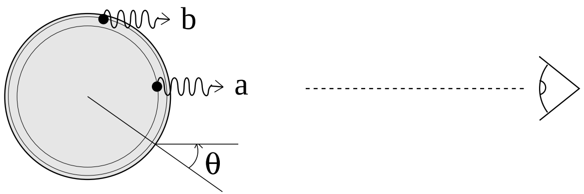
Eddington-Barbier approximation

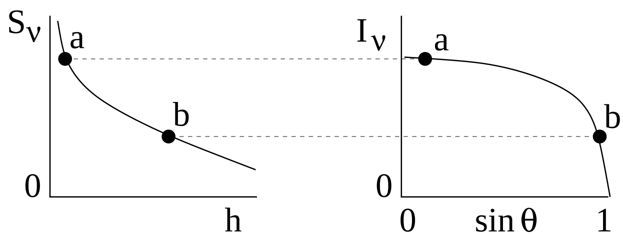
$$\boxed{I_\nu(0,\mu) \approx S_\nu(\tau_\nu\!=\!\mu)}$$



$$\tau_\lambda = -\int \alpha_\lambda\,\mathrm{d}h$$





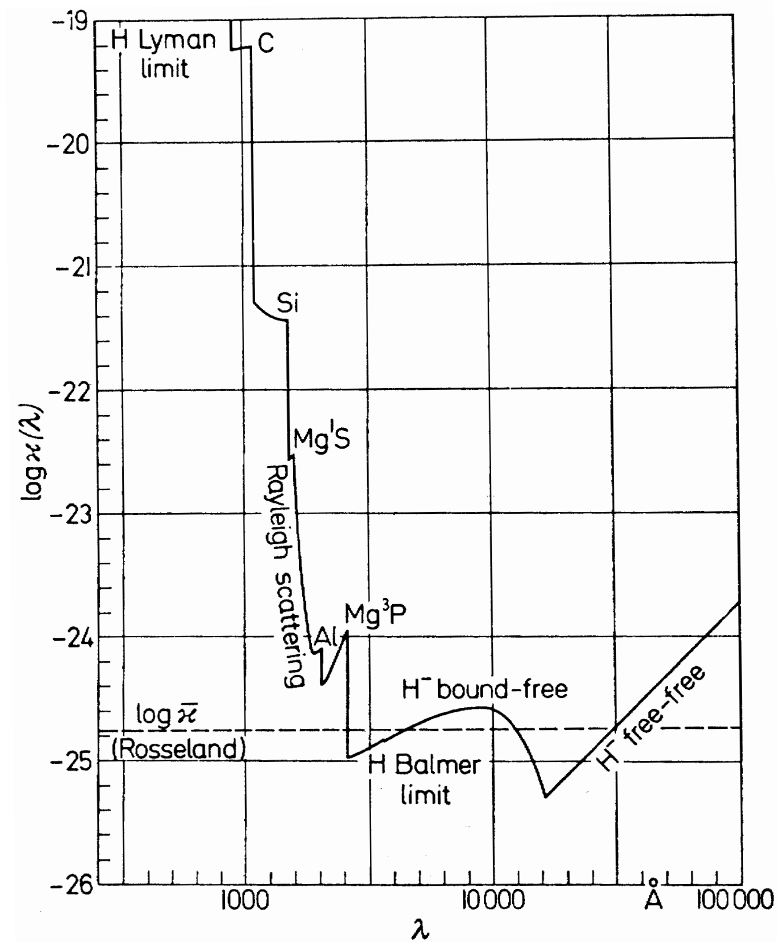


CONTINUOUS OPACITY IN THE SOLAR PHOTOSPHERE

Figure from E. Böhm-Vitense

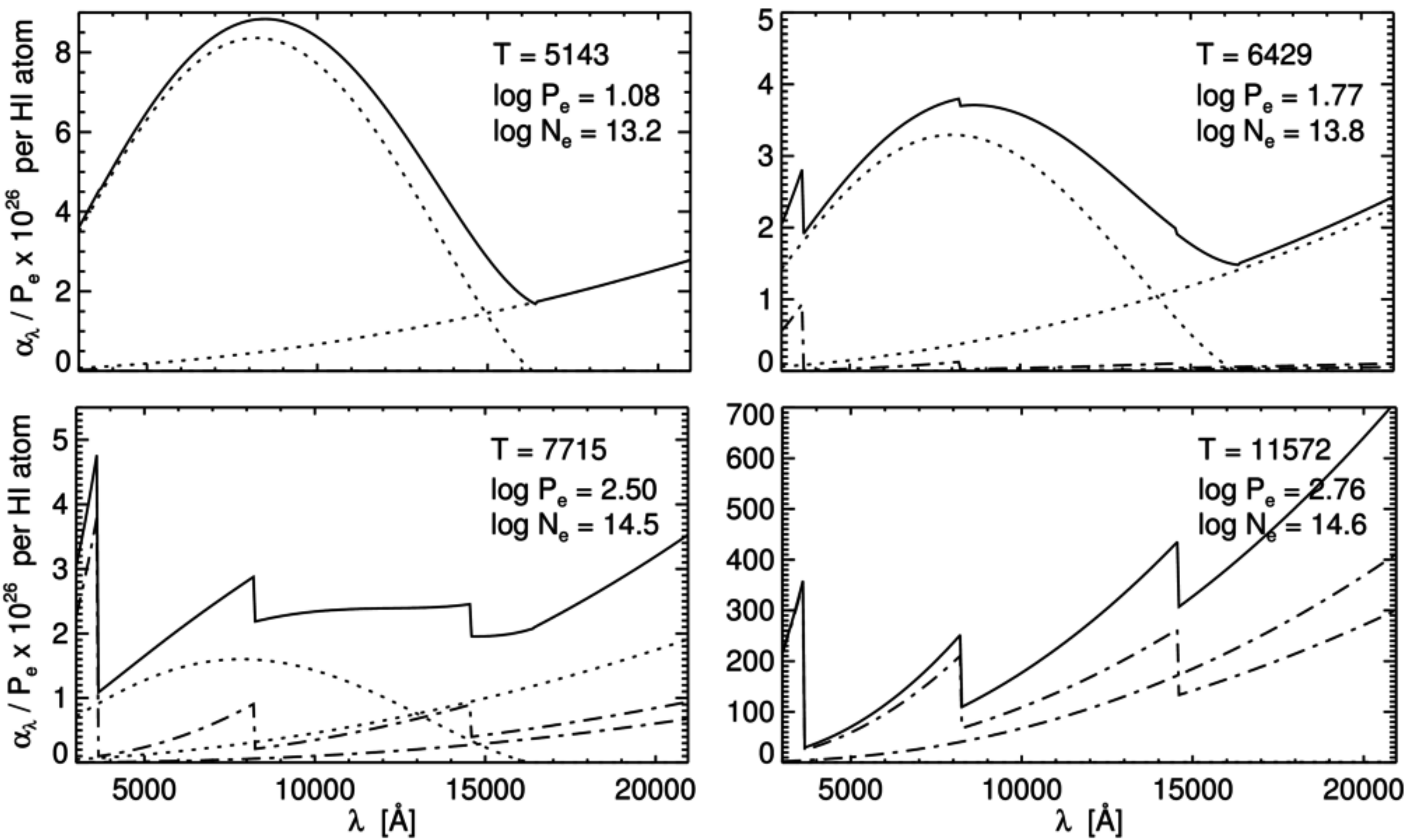
- *bound-free*
  - optical, near-infrared: H^-
  - UV: Si I, Mg I, Al I, Fe I
    (electron donors for H^-)
  - EUV: H I Lyman; He I, He II
- *free-free*
  - infrared, sub-mm: H^-
  - radio: H I
- *electron scattering*
  - Thomson scattering (large height)
  - Rayleigh scattering (near-UV)
- *Rosseland average*

$$\frac{1}{\overline{\kappa}} = \int_0^\infty \frac{1}{\kappa_\nu} \frac{\mathrm{d}B_\nu/\mathrm{d}T}{\mathrm{d}B/\mathrm{d}T}\,\mathrm{d}\nu$$



CONTINUOUS EXTINCTION H I bf, H I ff, H^- bf, H^- ff, TOTAL

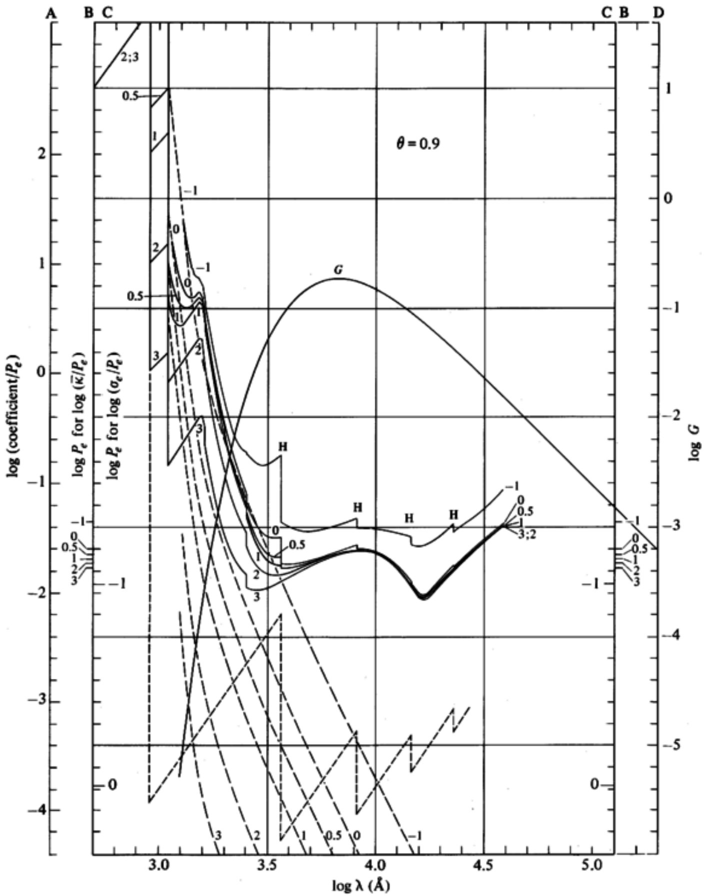
after Figures 8.5 of Gray (2005), without He and with Fig. 8.5a corrected



ERIKA [BOEHM-] VITENSE THESIS DIAGRAMS

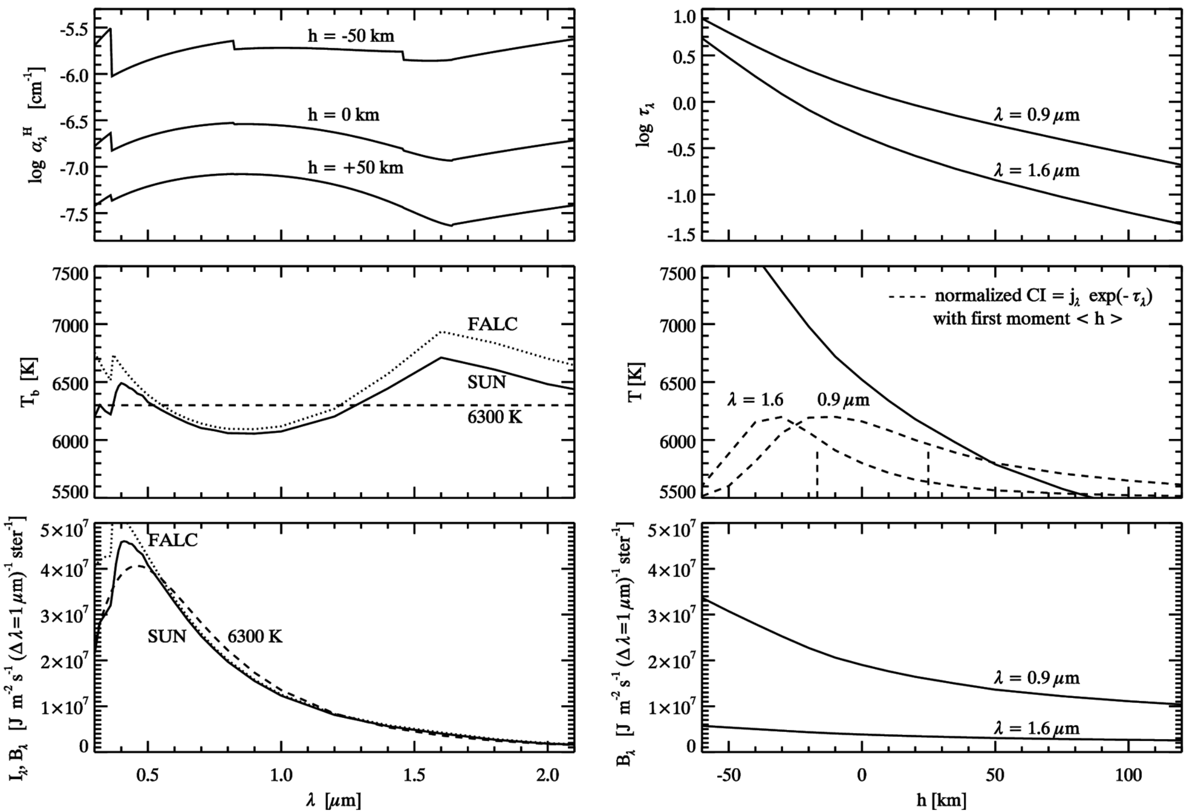
Vitense 1951ZA.....28...81V *Novotny* 1973itsa.book.....N

explanation: caption Fig. 8.6 RTSA and Exercise 10 RTSA



SOLAR OPTICAL AND NEAR-INFRARED CONTINUUM FORMATION

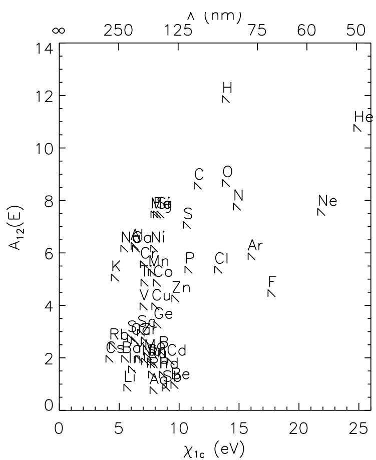
Assumed: LTE, opacity only from H I bf+ff and H^- bf+ff, FALC model atmosphere
Solar disk-center continuum: from Allen, *"Astrophysical Quantities"*, 1976

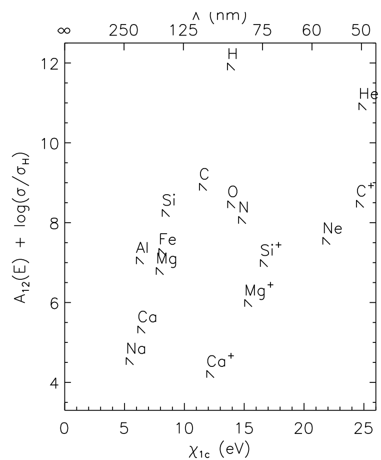


Does the Eddington-Barbier approximation hold?

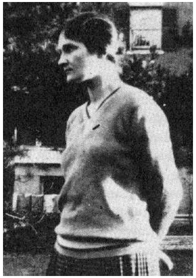
ABUNDANCES AND IONIZATION ENERGIES

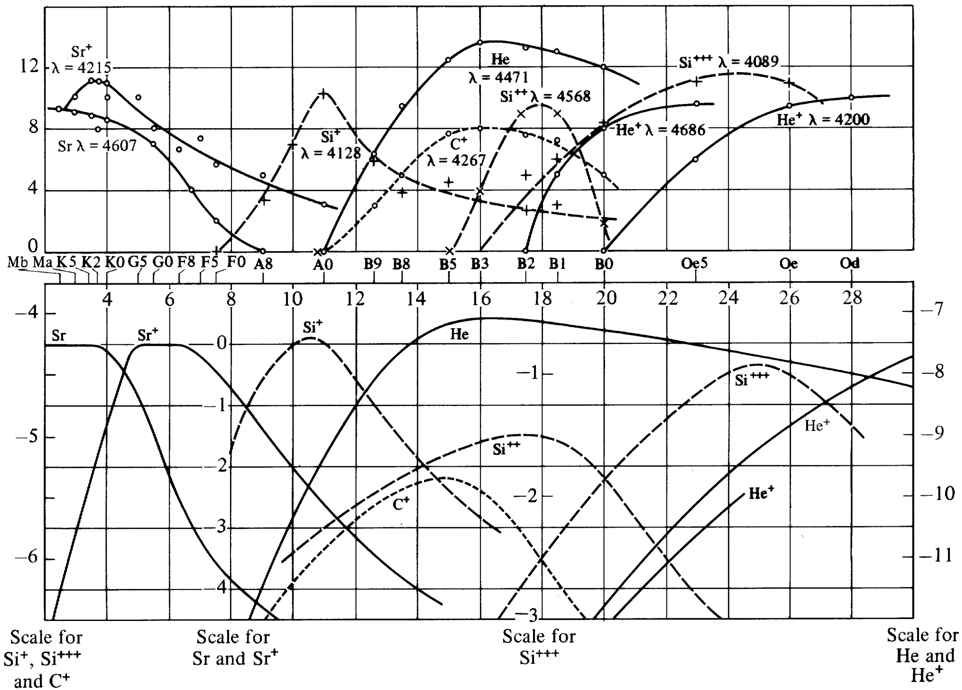
nr.	element	solar abundance	χ_1	χ_2	χ_3	χ_4
1	H	1	13.598	–	–	–
2	He	7.9×10^{-2}	24.587	54.416	–	–
6	C	3.2×10^{-4}	11.260	24.383	47.887	64.492
7	N	1.0×10^{-4}	14.534	29.601	47.448	77.472
8	O	6.3×10^{-4}	13.618	35.117	54.934	77.413
11	Na	2.0×10^{-6}	5.139	47.286	71.64	98.91
12	Mg	2.5×10^{-5}	7.646	15.035	80.143	109.31
13	Al	2.5×10^{-6}	5.986	18.826	28.448	119.99
14	Si	3.2×10^{-5}	8.151	16.345	33.492	45.141
20	Ca	2.0×10^{-6}	6.113	11.871	50.91	67.15
26	Fe	3.2×10^{-5}	7.870	16.16	30.651	54.8
38	Sr	7.1×10^{-10}	5.695	11.030	43.6	57





CECILIA PAYNE'S POPULATION CURVES





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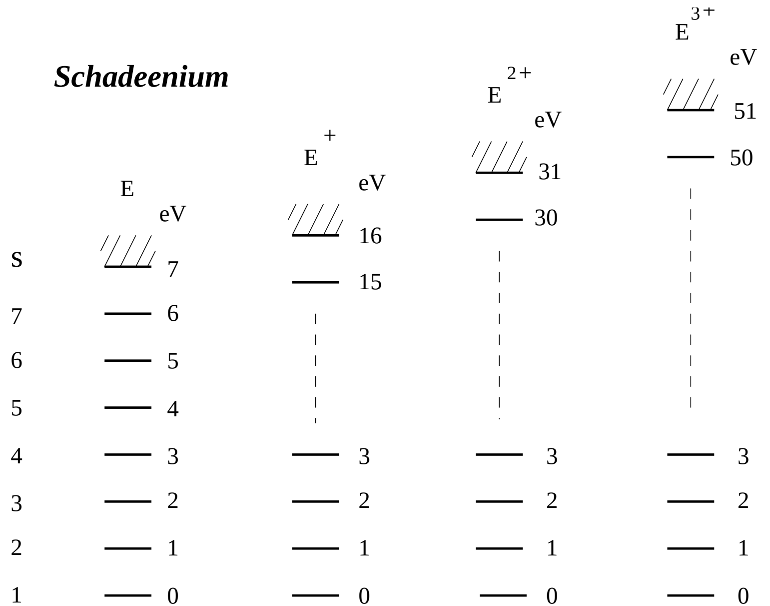
SAHA–BOLTZMANN POPULATIONS

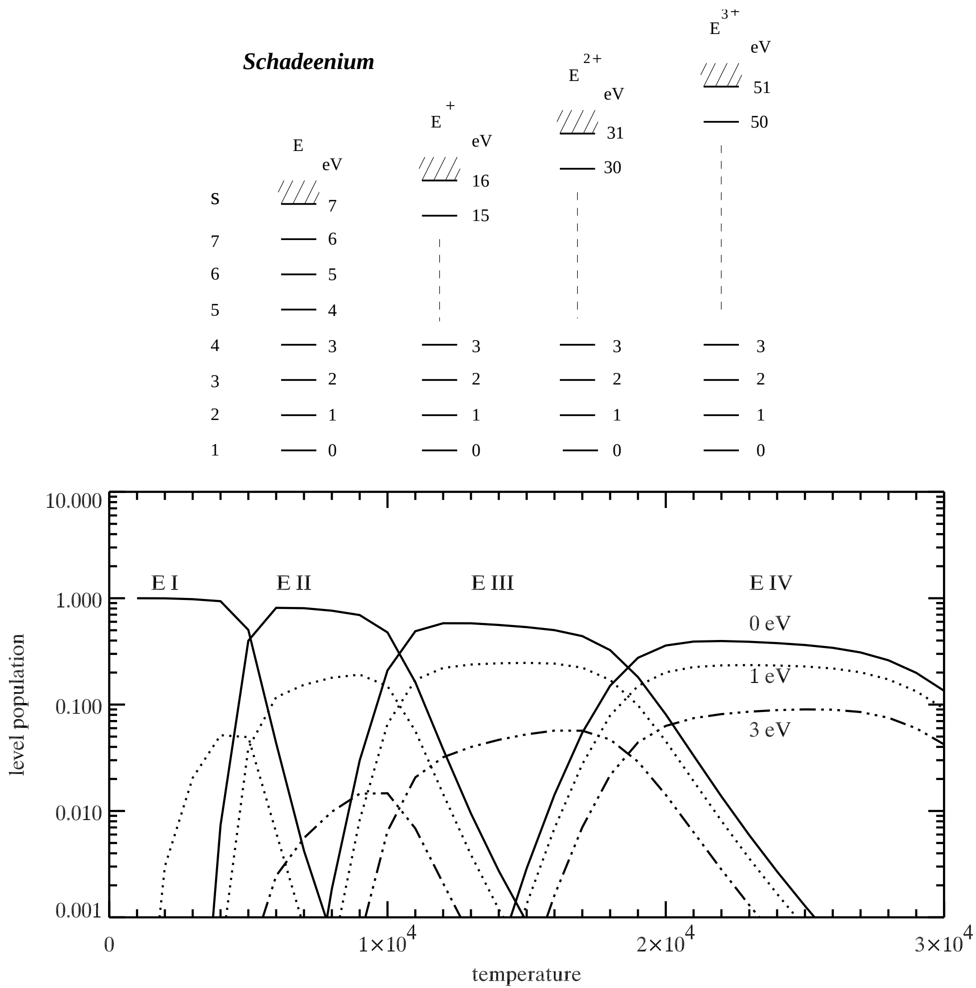


Boltzmann distribution per ionization stage: $\frac{n_{r,s}}{N_r} = \frac{g_{r,s}}{U_r}\, \mathrm{e}^{-\chi_{r,s}/kT}$

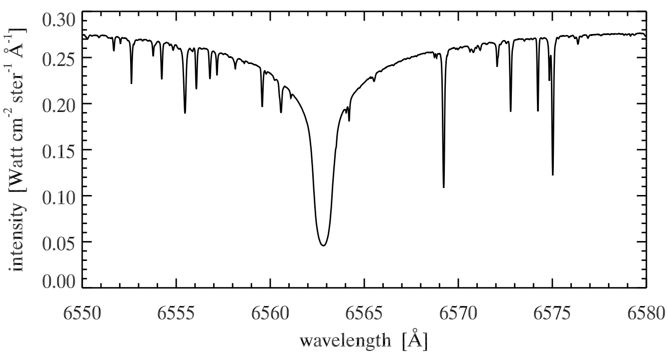
partition function: $U_r \equiv \sum_s g_{r,s}\, \mathrm{e}^{-\chi_{r,s}/kT}$

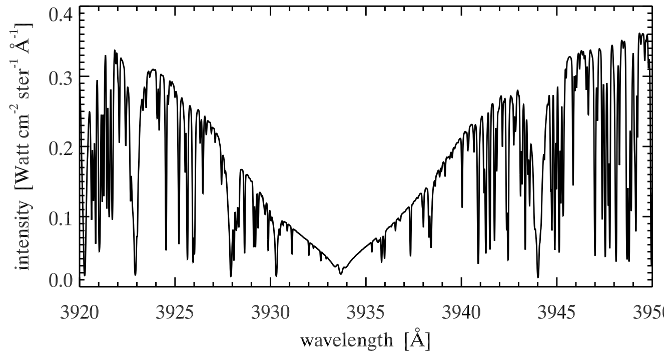
Saha distribution over ionization stages: $\frac{N_{r+1}}{N_r} = \frac{1}{N_\mathrm{e}} \frac{2\,U_{r+1}}{U_r} \left(\frac{2\pi m_\mathrm{e} kT}{h^2}\right)^{3/2} \mathrm{e}^{-\chi_r/kT}$

SAHA–BOLTZMANN FOR SCHADEENIUM

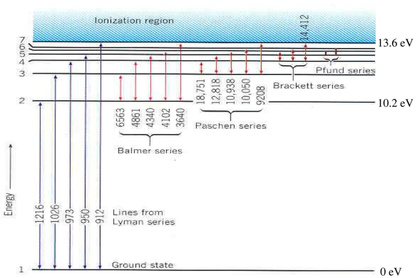


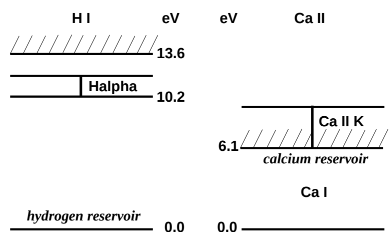
H-α AND Ca II K IN THE SOLAR SPECTRUM

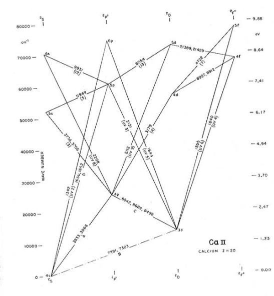




solar abundance ratio: Ca/H = 2×10^{-6}







Assuming LTE at $T = 5000$ K, $P_e = 10^2$ dyne cm^{-2}:

Boltzmann H I: $\dfrac{n_2}{n_1} = 4.2 \times 10^{-10}$ Saha Ca II: $\dfrac{N_{\rm Ca\,II}}{N_{\rm Ca}} \approx 1$ $\dfrac{\text{Ca II}\,(n=1)}{\text{H I}\,(n=2)} = 8 \times 10^3$

LTE EXTINCTION FOR Ca II H AND Hα IN FALC

Leenaarts et al. *2006A&A...449.1209L*

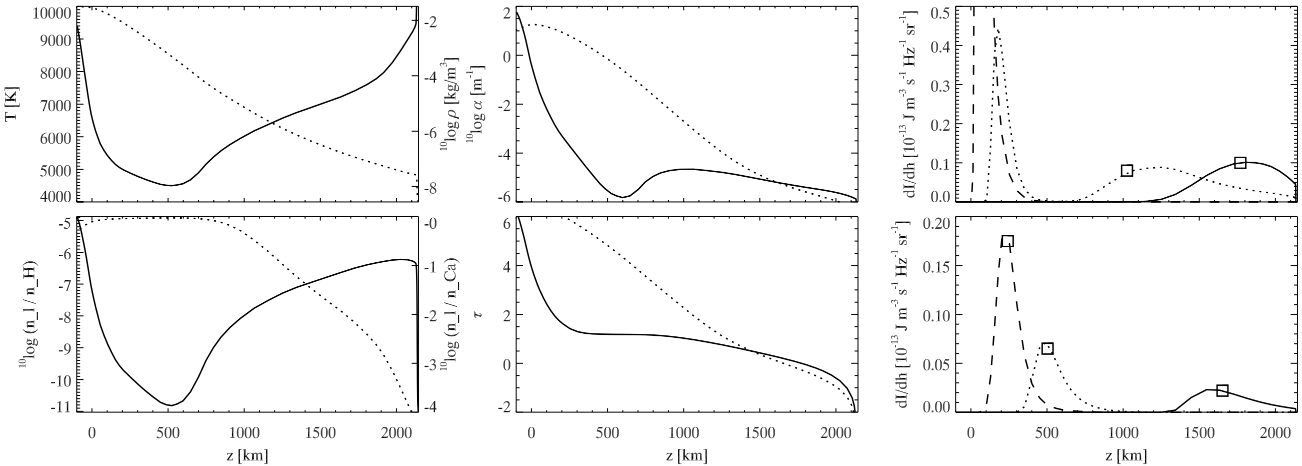
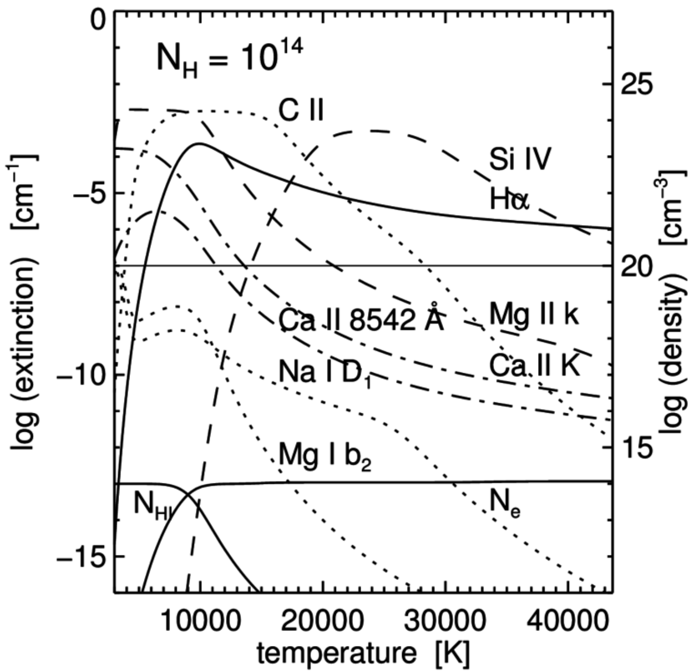


Fig. 6. Illustration explaining the difference between Hα and Ca II H wing formation. *Top left*: FALC temperature (lefthand scale, solid) and density (righthand scale, dotted). *Bottom left*: number density of the lower level of the line relative to the total number density of the atomic species, respectively for Hα (lefthand scale, solid) and for Ca II H (righthand scale, dotted). *Top center*: line center extinction coefficient for Hα (solid) and Ca II H (dotted). *Bottom center*: line center optical depth for Hα (solid) and Ca II H (dotted). *Top right*: intensity contribution function for $\Delta\lambda = 0$ (solid), -0.038 (dotted), and -0.084 nm (dashed) from line center in Hα. Squares indicate the $\tau = 1$ height. *Bottom right*: the same for Ca II H, at $\Delta\lambda = 0$ (solid), -0.024 (dotted), and -0.116 nm (dashed) from line center.

SOLAR SAHA-BOLTZMANN EXTINCTION OF STRONG LINES



- parcel of solar-composition gas with given total hydrogen density
- bottom curves: neutral hydrogen density, electron density
- other curves: line extinction for Saha-Bolzmann lower-level populations
- horizontal line: $\tau = 1$ thickness for a 100-km slab
- lower $N_{\rm H}$: curves shift but patterns remain similar
- note: Hα hot-gas champion, Hα – 8542 crossover, Mg I b_2 – Na I D_1 crossover

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LINE BROADENING

Doppler broadening = line-of-sight Maxwell component [+ "microturbulence"]: Gaussian

$$\Delta\nu_{\rm D} \equiv \frac{\nu_0}{c}\sqrt{\frac{2kT}{m}\,[+\xi^2_{\rm micro}]} \qquad \varphi(\nu-\nu_0) = \frac{1}{\sqrt{\pi}\,\Delta\nu_{\rm D}}\,{\rm e}^{-(\Delta\nu/\Delta\nu_{\rm D})^2}$$

natural broadening = radiative damping from uncertainty relation: Lorentzian

$$\gamma^{\rm rad} = \gamma^{\rm rad}_l + \gamma^{\rm rad}_u = \sum_{i<l} A_{li} + \sum_{i<u} A_{ui} \qquad \psi(\nu-\nu_0) = \frac{\gamma^{\rm rad}/4\pi^2}{(\nu-\nu_0)^2 + (\gamma^{\rm rad}/4\pi)^2}$$

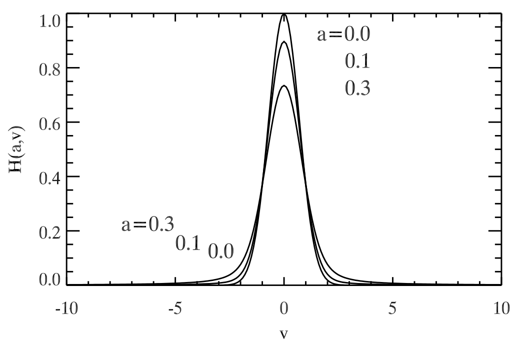
collisional damping: impact (Lorentzian) or quasistatic approximation (Holtsmark)

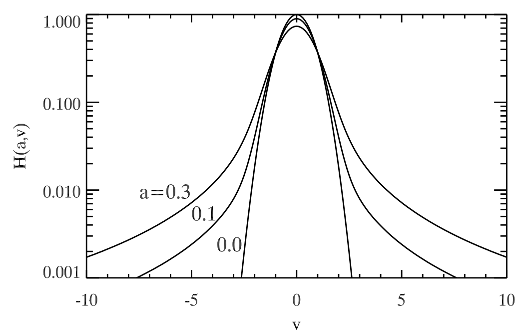
metal lines: Van der Waals, Lorentzian with $\gamma^{\rm total} = \gamma^{\rm rad} + \gamma^{\rm coll}$

hydrogen lines: linear Stark + resonance, Holtsmark

line extinction coefficient shape = convolution Gaussian $\otimes$ Lorentzian = Voigt function

$$\phi(\nu) = \frac{1}{\sqrt{\pi}\Delta\nu_{\rm D}}\,H(a,v) \qquad v \equiv \frac{\nu-\nu_0}{\Delta\nu_{\rm D}} \qquad a \equiv \frac{\gamma^{\rm total}}{4\pi\Delta\nu_{\rm D}} \qquad H(a,v) \equiv \frac{a}{\pi}\int_{-\infty}^{+\infty}\frac{{\rm e}^{-y^2}}{(v-y)^2+a^2}{\rm d}y$$





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LTE LINES

Voigt function

$$v \equiv \frac{\nu-\nu_0}{\Delta\nu_\mathrm{D}} \qquad a \equiv \frac{\gamma}{4\pi\Delta\nu_\mathrm{D}} \qquad H(a,v) \equiv \frac{a}{\pi}\int_{-\infty}^{+\infty} \frac{\mathrm{e}^{-y^2}}{(v-y)^2+a^2}\,\mathrm{d}y$$

area in ν: $\sqrt{\pi}\Delta\nu_\mathrm{D}$

line extinction coefficient in LTE

$$\sigma_\nu^l = \frac{h\nu}{4\pi}\,B_{lu}\,\phi(\nu) = \frac{\pi e^2}{m_\mathrm{e}c}\,f_l\,\phi(\nu) = \frac{\sqrt{\pi}\,e^2\,f_l}{m_\mathrm{e}c\,\Delta\nu_\mathrm{D}}\,H(a,v)$$

$$A_{ul} = 6.67\times 10^{13}\,\frac{g_l}{g_u}\,\frac{f_{lu}}{\lambda^2}\ \mathrm{s}^{-1}\ (\lambda \text{ in nm})$$

$$\alpha_\nu^l = \sigma_\nu^l\,n_l^\mathrm{LTE}\,(1-\mathrm{e}^{-h\nu/kT}) = \sigma_\nu^l\,\frac{n_i}{\sum n_i}\,\frac{n_{ij}}{n_i}\,\frac{n_{ijk}}{n_{ij}}\,(1-\mathrm{e}^{-h\nu/kT})$$

i, j, k species, stage, lower level

abundance, Saha, Boltzmann

use continuum optical depth scale as reference

$$\eta_\nu \equiv \alpha_\nu^l/\alpha_\nu^c \qquad \mathrm{d}\tau_\nu = \mathrm{d}\tau_\nu^c + \mathrm{d}\tau_\nu^l = (1+\eta_\nu)\,\mathrm{d}\tau_\nu^c$$

emergent intensity at disk center in LTE

$$I_\nu(0,1) = \int_0^\infty B_\nu\,\exp(-\tau_\nu)\,\mathrm{d}\tau_\nu = \int_0^\infty (1+\eta_\nu)\,B_\nu\,\exp\left(-\int_0^{\tau_\nu^c}(1+\eta_\nu)\,\mathrm{d}t_\nu^c\right)\,\mathrm{d}\tau_\nu^c$$

Eddington-Barbier

$$I_\nu(0,1) \approx B_\nu\,(T\,[\tau_\nu\!=\!1]) = B_\nu\,(T\,[\tau_\nu^c\!=\!1/(1\!+\!\eta_\nu)])$$

SOLAR SPECTRUM FORMATION: THEORY

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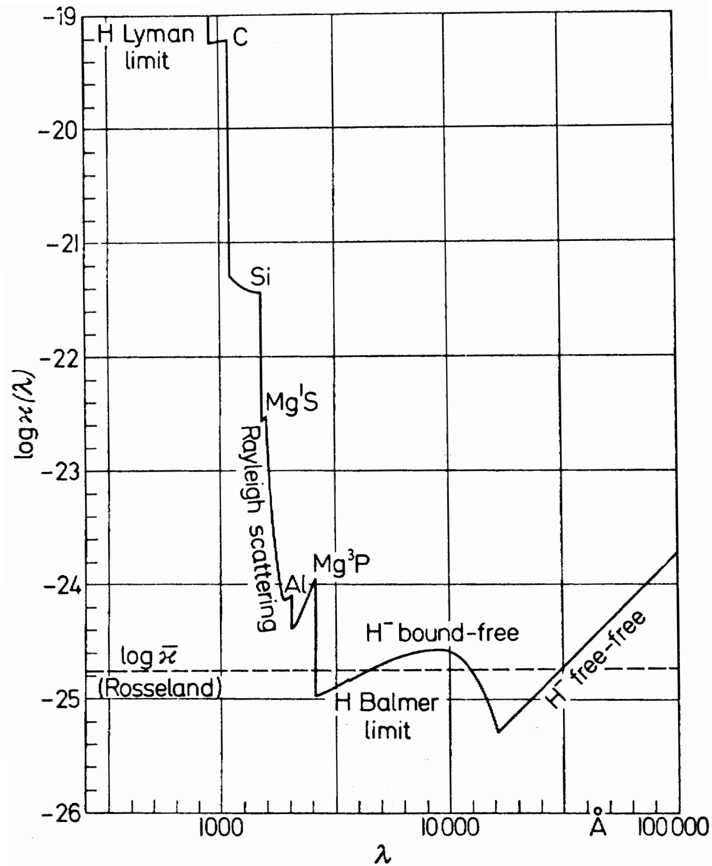
SOLAR ATMOSPHERE RADIATIVE PROCESSES

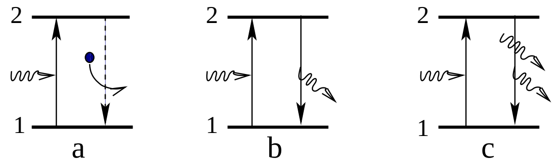
- *bound-bound* – κ_ν, S_ν: CE, LTE, NLTE, PRD, NSE?
  - neutral atom transitions
  - ion transitions
  - molecule transitions
- *bound-free* – same except always CRD
  - H^- optical, near-infrared
  - H I Balmer, Lyman; He I, He II
  - Fe I, Si I, Mg I, Al I $=$ electron donors
- *free-free* – $S_\nu = B_\nu$
  - H^- infrared, sub-mm
  - H I mm, radio
- *electron scattering* – $S_\nu = J_\nu$
  - Thomson scattering
  - Rayleigh scattering
- *collective* – p.m.
  - cyclotron, synchrotron radiation
  - plasma radiation



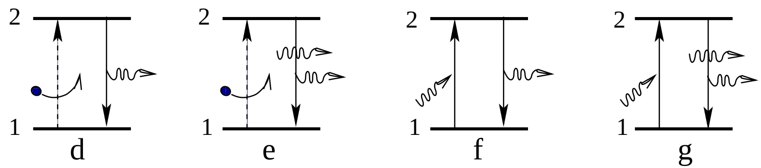
E. Böhm-Vitense

BOUND-BOUND EQUILIBRIA

line extinction:

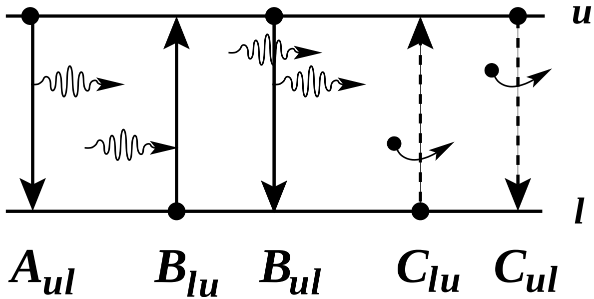


line emissivity:



- *LTE* $=$ *large collision frequency* – interior, low photosphere
  - up: mostly collisional $=$ thermal creation (d + e)
  - down: mostly collisional $=$ large destruction probability (a)
  - photon travel: “honorary gas particles” or negligible leak
- *NLTE, NSE* $=$ *statistical equilibrium or time-dependent* – chromosphere, “TR”
  - photon travel: non-local impinging (pumping), loss (suction)
  - two-level scattering: complete/partial/angle redistribution
  - multi-level: photon conversion, sensitivity transcription
- *CE* $=$ *coronal equilibrium* $=$ *thin tenuous* – coronal EUV
  - up: only collisional $=$ thermal creation (only d)
  - down: only spontaneous (only d)
  - photon travel: escape / drown / scatter bf H I, He I, He II

BOUND-BOUND PROCESSES AND EINSTEIN COEFFICIENTS



Spontaneous deexcitation

$A_{ul} \equiv$ transition probability for spontaneous deexcitation from state u to state l per sec per particle in state u

Radiative excitation

$B_{lu}\overline{J}^{\varphi}_{\nu_0} \equiv$ number of radiative excitations from state l to state u per sec per particle in state l

Induced deexcitation

$B_{ul}\overline{J}^{\chi}_{\nu_0} \equiv$ number of induced radiative deexcitations from state u to state l per sec per particle in state u

Collisional excitation and deexcitation

$C_{lu} \equiv$ number of collisional excitations from state l to state u per sec per particle in state l

$C_{ul} \equiv$ number of collisional deexcitations from state u to state l per sec per particle in state u

LINE SOURCE FUNCTION

RTSA 2.3.1, 2.3.2, 2.6.1

Monochromatic bb rates expressed in Einstein coefficients (per steradian, as intensity)

$n_u A_{ul} \chi(\nu)/4\pi$	$n_u B_{ul} I_\nu \psi(\nu)/4\pi$	$n_l B_{lu} I_\nu \phi(\nu)/4\pi$	$n_u C_{ul}$ $n_l C_{lu}$
spontaneous emission	stimulated emission	radiative excitation	collisional (de-)excitation

Einstein relations

$$g_u B_{ul} = g_l B_{lu} \qquad (g_u/g_l) A_{ul} = (2h\nu^3/c^2)\, B_{lu} \qquad C_{ul}/C_{lu} = (g_l/g_u) \exp(E_{ul}/kT)$$

required for TE detailed balancing with $I_\nu = B_\nu$, but hold universally

General line source function

$$j_\nu = \frac{h\nu}{4\pi}\, n_u\, A_{ul}\, \chi(\nu) \qquad \alpha_\nu = \frac{h\nu}{4\pi}\, [n_l\, B_{lu} \phi(\nu) - n_u\, B_{ul} \psi(\nu)] \qquad S_l = \frac{n_u\, A_{ul}\, \chi(\nu)}{n_l\, B_{lu} \phi(\nu) - n_u\, B_{ul} \psi(\nu)}$$

Simplified line source function

CRD: $\chi(\nu) = \psi(\nu) = \phi(\nu)$

$$S_l = \frac{n_u\, A_{ul}}{n_l\, B_{lu} - n_u\, B_{ul}} = \frac{2h\nu^3}{c^2} \frac{1}{\frac{g_u n_l}{g_l n_u} - 1}$$

Boltzmann: $S_l = B_\nu(T)$

FORMAL TEMPERATURES

RTSA 2.6.2

Excitation temperature

$$\frac{n_u}{n_l} \equiv \frac{g_u}{g_l}\,\mathrm{e}^{-h\nu/kT_{\mathrm{exc}}} \qquad S^l_{\nu_0} = \frac{2h\nu_0^3}{c^2}\frac{1}{\frac{g_u n_l}{g_l n_u} - 1} = \frac{2h\nu_0^3}{c^2}\frac{1}{\mathrm{e}^{h\nu_0/kT_{\mathrm{exc}}} - 1} = B_{\nu_0}(T_{\mathrm{exc}})$$

Ionization temperature

$$S^{\mathrm{bf}}_\nu \equiv \frac{2h\nu^3}{c^2}\frac{1}{\mathrm{e}^{h\nu/kT_{\mathrm{ion}}} - 1} = B_\nu(T_{\mathrm{ion}})$$

Radiation temperature

$$B_\nu(T_{\mathrm{rad}}) \equiv J_\nu$$

Brightness temperature

$$B_\nu(T_{\mathrm{b}}) \equiv I_\nu$$

Effective temperature

$$\pi B(T_{\mathrm{eff}}) \equiv \sigma T_{\mathrm{eff}}^4 = \mathcal{F}_{\mathrm{surface}}$$

POPULATION DEPARTURE COEFFICIENTS

RTSA 2.6.2

Population departure coefficients

$$b_l = n_l / n_l^{\rm LTE} \qquad b_u = n_u / n_u^{\rm LTE}$$

Line source function

$$S_\nu^l = \frac{2h\nu^3}{c^2} \frac{\psi/\varphi}{\dfrac{b_l}{b_u}\,{\rm e}^{h\nu/kT} - \dfrac{\chi}{\varphi}} \qquad \text{CRD:}\;\; S_{\nu_0}^l = \frac{2h\nu_0^3}{c^2} \frac{1}{\dfrac{b_l}{b_u}\,{\rm e}^{h\nu_0/kT} - 1}, \qquad \text{Wien:}\;\; S_{\nu_0}^l \approx \frac{b_u}{b_l} B_{\nu_0}$$

Monochromatic line extinction coefficient

$$\alpha_\nu^l = \frac{h\nu}{4\pi}\, b_l\, n_l^{\rm LTE} B_{lu} \varphi(\nu-\nu_0) \left[1 - \frac{b_u\, n_u^{\rm LTE}\, B_{ul}\, \chi}{b_l\, n_l^{\rm LTE}\, B_{lu}\, \varphi}\right] = \frac{h\nu}{4\pi}\, b_l\, n_l^{\rm LTE} B_{lu} \varphi(\nu-\nu_0) \left[1 - \frac{b_u}{b_l}\frac{\chi}{\varphi}\,{\rm e}^{-h\nu/kT}\right]$$

$$= b_l\, n_l^{\rm LTE} \sigma_\nu^l \left[1 - \frac{b_u}{b_l}\frac{\chi}{\varphi}\,{\rm e}^{-h\nu/kT}\right] = \frac{\pi e^2}{m_{\rm e} c}\, b_l\, n_l^{\rm LTE} f_{lu} \varphi(\nu-\nu_0) \left[1 - \frac{b_u}{b_l}\frac{\chi}{\varphi}\,{\rm e}^{-h\nu/kT}\right]$$

$$\text{Wien:}\;\; \alpha_\nu^l \approx b_l \left[\alpha_\nu^l\right]_{\rm LTE}$$

Total line extinction coefficient

$$\alpha_{\nu_0}^l = \frac{h\nu_0}{4\pi}\, b_l\, n_l^{\rm LTE} B_{lu} \left[1 - \frac{b_u}{b_l}\,{\rm e}^{-h\nu_0/kT}\right] = \frac{\pi e^2}{m_{\rm e} c}\, b_l\, n_l^{\rm LTE} f_{lu} \left[1 - \frac{b_u}{b_l}\,{\rm e}^{-h\nu_0/kT}\right] \approx b_l \left[\alpha_{\nu_0}^l\right]_{\rm LTE}$$

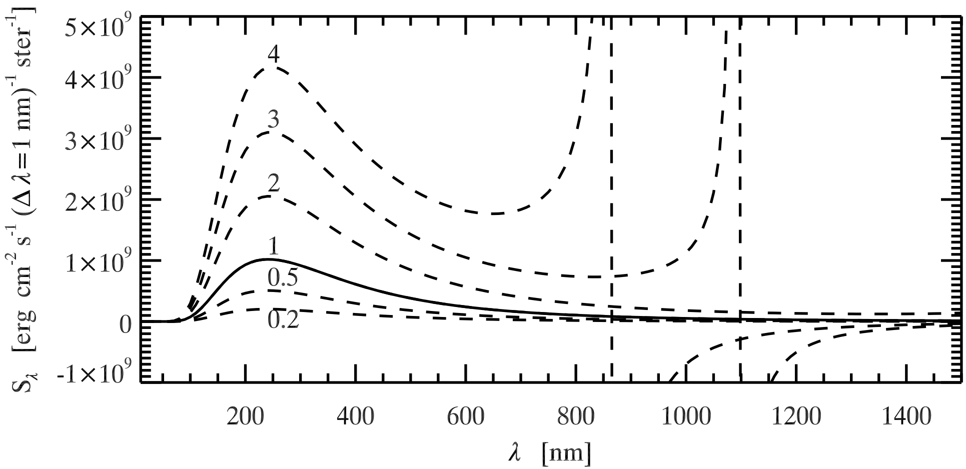
LASERING

RTSA 2.6.2

Laser regime for sufficient excess $b_u > b_l$

$$1 - (b_u/b_l)\exp(-h\nu_0/kT) < 0 \quad\Longrightarrow\quad \alpha^l_{\nu_0} < 0 \qquad S^l_{\nu_0} < 0$$

$$\frac{S^l_{\nu_0}}{B_{\nu_0}} = \frac{1 - \,\mathrm{e}^{-h\nu_0/kT}}{(b_l/b_u)\left[1 - (b_u/b_l)\,\mathrm{e}^{-h\nu_0/kT}\right]} = b_u \frac{\left[\alpha^l_{\nu_0}\right]_{\mathrm{LTE}}}{\alpha^l_{\nu_0}}$$



Wavelength variation of the NLTE source function for $T = 10\,000$ K and the specified ratios b_u/b_l. The NLTE source function scales with the Planck function (solid curve for $b_u/b_l = 1$) in the Wien part at left, but reaches the laser regime for large b_u/b_l in the Rayleigh-Jeans part at right.

STATISTICAL EQUILIBRIUM VERSUS NON-EQUILIBRIUM EVALUATION

RTSA 2.6.1

Statistical equilibrium equations for level j

$$n_j \sum_{j\neq i}^{N} R_{ji} = \sum_{j\neq i}^{N} n_j R_{ij} \qquad R_{ji} = A_{ji} + B_{ji}\overline{J_{ji}} + C_{ji} \qquad \overline{J_{ji}} \equiv \frac{1}{4\pi}\int_0^{4\pi}\int_0^{\infty} I_\nu\,\phi(\nu)\,\mathrm{d}\nu\,\mathrm{d}\Omega$$

time-independent population — bb rates per particle in j — total (= mean) mean intensity for CRD

Transport equation in differential form

$$\mu\frac{\mathrm{d}I_\nu}{\mathrm{d}\tau_\nu} = I_\nu - S_\nu$$

Transport equation in integral form

$$I_\nu^-(\tau_\nu,\mu) = -\int_0^{\tau_\nu} S_\nu(t_\nu)\,\mathrm{e}^{-(t_\nu-\tau_\nu)/\mu}\,\mathrm{d}t_\nu/\mu$$

$$I_\nu^+(\tau_\nu,\mu) = +\int_{\tau_\nu}^{\infty} S_\nu(t_\nu)\,\mathrm{e}^{-(t_\nu-\tau_\nu)/\mu}\,\mathrm{d}t_\nu/\mu$$

1D (“plane-parallel”) SE: require in $=$ out, solve these coupled equations for all wavelengths and levels of all pertinent transitions for a sufficient number of μ angles at all heights

3D non-E: evaluate the net rates for all wavelengths and levels of all pertinent transitions for a sufficient number of ϕ and ψ angles at all (x,y,z) locations as a function of time. If energetically important, couple back into the energy equation in the simulation

SOLAR SPECTRUM FORMATION: THEORY

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start: dawn of astrophysics exercises literature 101-intro

basics: basic quantities flux intensity conservation exam constant S_ν plane-atmosphere RT EB CF+RF formation cartoons E-B exam $\mathbf{\Lambda}(S)$

LTE 1D static: Planck EB-line-limb continuous opacity electron donors Saha-Boltzmann line broadening LTE line equations

NLTE descriptions: solar radiation processes bb equilibria Einstein coefficients line source function formal temperatures departure coefficients lasering population + transport equations

scattering: 2-level atoms sharp atom CZ demo scattering equations results

refinements: partial redistribution multi-level detours radiative cooling balancing $\mathbf{\Lambda}$ iteration

course summary: all bb pairs NLTE line cartoon equation summary key equations scattering cont & line NLTE summary cartoon homework

course finish: H I exam moral conclusion

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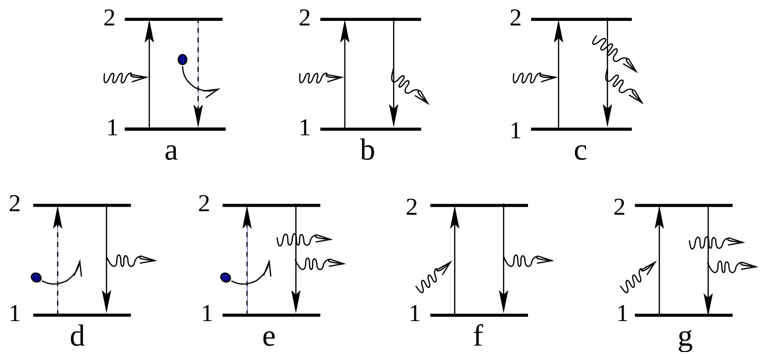
TWO-LEVEL TRANSPORT

RTSA 3.4

All two-level process pairs involving a beam photon (to the right)



Sharp-line two-level atom = monochromatic “redistribution”

$$\frac{\mathrm{d}I_{\nu_0}}{\mathrm{d}s} = \frac{h\nu_0}{4\pi}\, n_1 \Bigg[\underbrace{-B_{12} I_{\nu_0} \frac{C_{21}}{P_{21}}}_{\text{(a)}} \underbrace{- B_{12} I_{\nu_0} \frac{A_{21}}{P_{21}}}_{\text{(b)}} \underbrace{- B_{12} I_{\nu_0} \frac{B_{21} J_{\nu_0}}{P_{21}}}_{\text{(c)}}$$

$$+ \underbrace{C_{12} \frac{A_{21}}{P_{21}}}_{\text{(d)}} + \underbrace{C_{12} \frac{B_{21} I_{\nu_0}}{P_{21}}}_{\text{(e)}} + \underbrace{B_{12} J_{\nu_0} \frac{A_{21}}{P_{21}}}_{\text{(f)}} + \underbrace{B_{12} J_{\nu_0} \frac{B_{21} I_{\nu_0}}{P_{21}}}_{\text{(g)}} \Bigg]$$

TWO-LEVEL SOURCE FUNCTION

sharp-line atom derivation: RTSA 3.4

Collisional destruction probability per extinction

$$\varepsilon_{\nu_0} \equiv \frac{\alpha^{\mathrm{a}}_{\nu_0}}{\alpha^{\mathrm{s}}_{\nu_0} + \alpha^{\mathrm{a}}_{\nu_0}} = \frac{C_{21}}{A_{21}/[1-\exp(-h\nu_0/kT)] + C_{21}} = \frac{C_{21}}{A_{21} + B_{21}B_{\nu_0} + C_{21}}$$

Alternate form

$$\varepsilon'_{\nu_0} \equiv \alpha^{\mathrm{a}}_{\nu_0}/\alpha^{\mathrm{s}}_{\nu_0} = \frac{\varepsilon_{\nu_0}}{1-\varepsilon_{\nu_0}} = \frac{C_{21}}{A_{21}}\,[1-\ \mathrm{e}^{-h\nu_0/kT}]$$

Line source function

$$S^l_{\nu_0} \equiv \frac{j^l_{\nu_0}}{\alpha^l_{\nu_0}} = (1-\varepsilon_{\nu_0})\,J_{\nu_0} + \varepsilon_{\nu_0} B_{\nu_0} = \frac{J_{\nu_0} + \varepsilon'_{\nu_0} B_{\nu_0}}{1+\varepsilon'_{\nu_0}}$$

Complete frequency redistribution

$$S^l_{\nu_0} = (1-\varepsilon_{\nu_0})\,\overline{J}^{\varphi}_{\nu_0} + \varepsilon_{\nu_0} B_{\nu_0} = \frac{\overline{J}^{\varphi}_{\nu_0} + \varepsilon'_{\nu_0} B_{\nu_0}}{1+\varepsilon'_{\nu_0}}$$

Frequency-independent, but beware

$$S^{\mathrm{tot}}_{\nu} = \frac{\alpha^l_{\nu} S^l_{\nu_0} + \alpha^c_{\nu} S^c_{\nu}}{\alpha^l_{\nu} + \alpha^c_{\nu}}$$

SUMMARY SCATTERING EQUATIONS

RTSA 4.1–4.3

Destruction probability

coherent: $\varepsilon_\nu \equiv \dfrac{\alpha_\nu^{\mathrm{a}}}{\alpha_\nu^{\mathrm{a}} + \alpha_\nu^{\mathrm{s}}}$ 2-level CRD: $\varepsilon_{\nu_0} \equiv \dfrac{\alpha_{\nu_0}^{\mathrm{a}}}{\alpha_{\nu_0}^{\mathrm{a}} + \alpha_{\nu_0}^{\mathrm{s}}} = \dfrac{C_{21}}{C_{21} + A_{21} + B_{21} B_{\nu_0}}$

Elastic scattering

coherent: $S_\nu = (1-\varepsilon_\nu) J_\nu + \varepsilon_\nu B_\nu$ 2-level CRD: $S_{\nu_0} = (1-\varepsilon_{\nu_0}) \overline{J}_{\nu_0} + \varepsilon_{\nu_0} B_{\nu_0}$

Schwarzschild equation and Lambda operator

$$J_\nu(\tau_\nu) \equiv \frac{1}{2} \int_{-1}^{+1} I_\nu(\tau_\nu, \mu)\, \mathrm{d}\mu = \frac{1}{2} \int_0^\infty S_\nu(t_\nu)\, E_1(|t_\nu - \tau_\nu|)\, \mathrm{d}t_\nu \equiv \mathbf{\Lambda}_{\tau_\nu}[S_\nu(t_\nu)]$$

surface: $J_\nu(0) \approx \frac{1}{2} S_\nu(\tau_\nu = 1/2)$ depth: $J_\nu(\tau_\nu) \approx S_\nu(\tau_\nu)$ diffusion: $J_\nu(\tau_\nu) \approx B_\nu(\tau_\nu)$

Scattering in an isothermal atmosphere with constant ε_ν

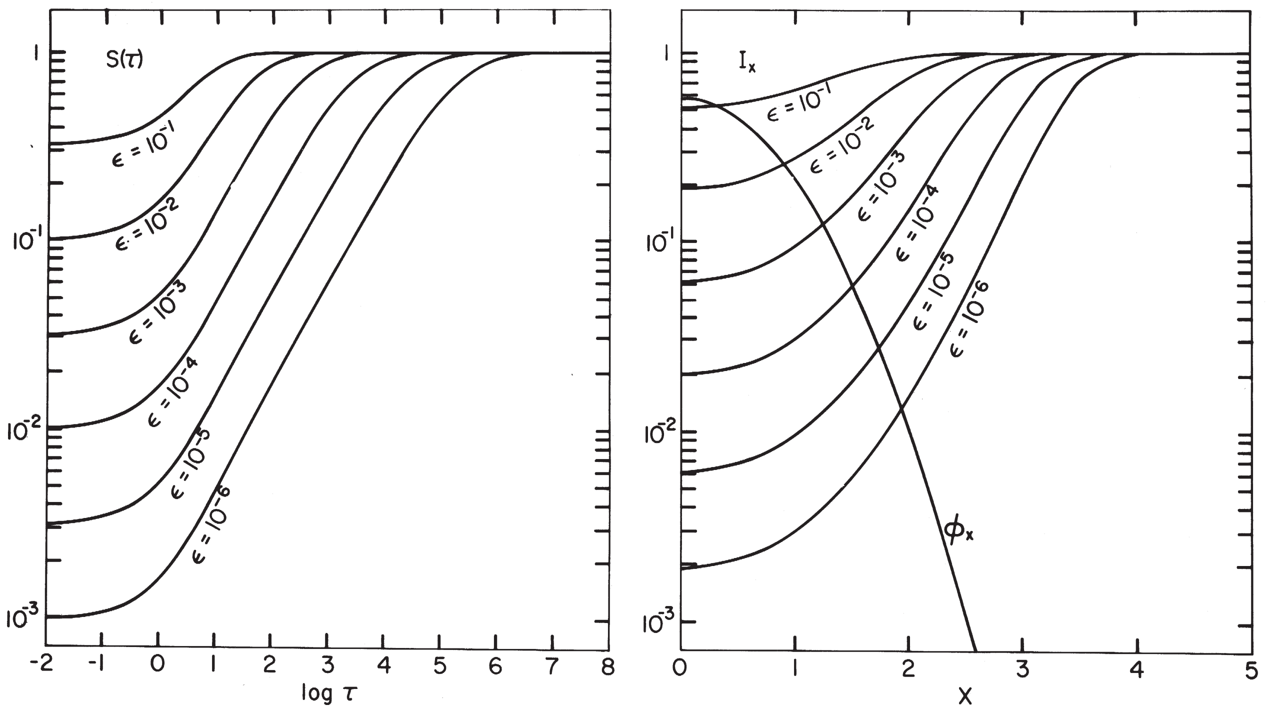
coherent: $S_\nu(0) = \sqrt{\varepsilon_\nu}\, B_\nu$ 2-level CRD: $S_{\nu_0}(0) = \sqrt{\varepsilon_{\nu_0}}\, B_{\nu_0}$

Thermalizaton depth

coherent: $\Lambda_\nu = 1/\varepsilon_\nu^{1/2}$ Gauss profile: $\Lambda_{\nu_0} \approx 1/\varepsilon_{\nu_0}$ Lorentz profile: $\Lambda_{\nu_0} \approx 1/\varepsilon_{\nu_0}^2$

CRD RESONANT SCATTERING IN AN ISOTHERMAL ATMOSPHERE

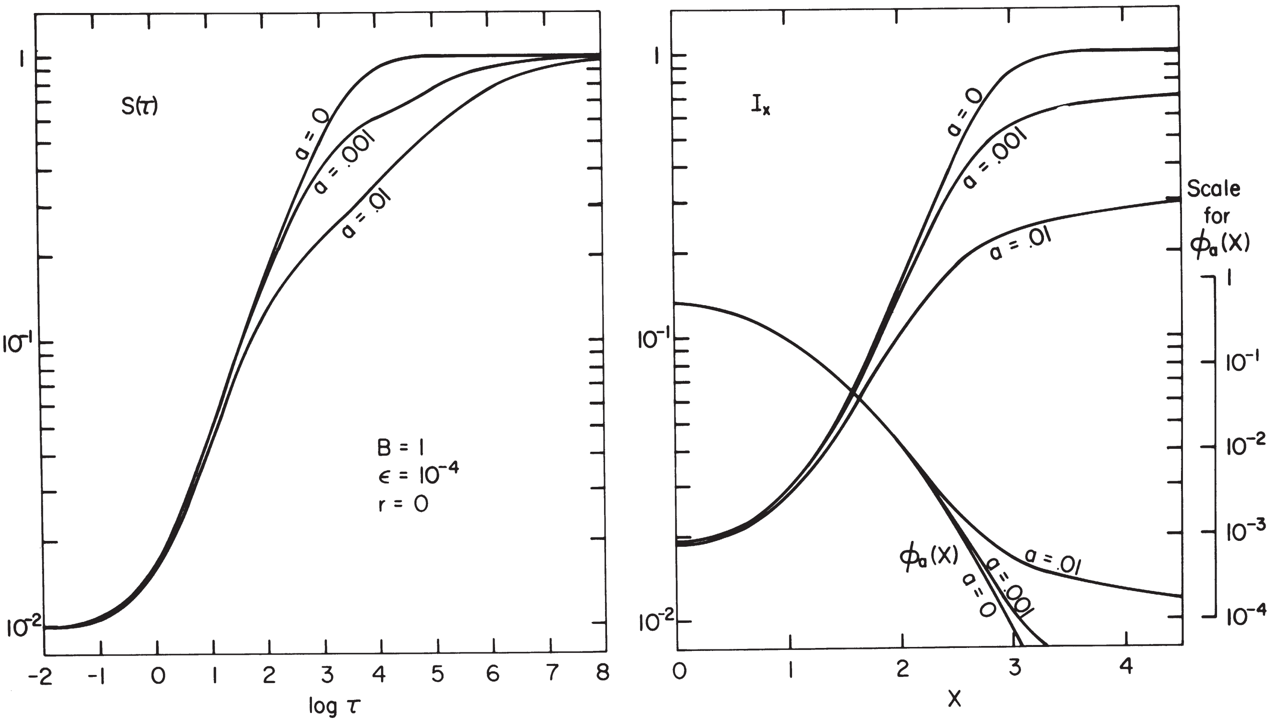
RTSA figure 4.12; from Avrett 1965SAOSR.174..101A



- left: S/B in a plane-parallel isothermal atmosphere with constant ε for complete redistribution. The curves illustrate the $\sqrt{\varepsilon}$ law and thermalization at $\Lambda \approx 1/\varepsilon$.
- right: corresponding emergent line profiles and Gaussian extinction profile shape ϕ (only the righthand halves; $x = \Delta\lambda/\Delta\lambda_{\rm D}$)

TWO-LEVEL SCATTERING FOR DIFFERENT LINE PROFILES

RTSA figure 4.12; from Avrett 1965SAOSR.174..101A



- left: S/B in a plane-parallel isothermal atmosphere with constant $\varepsilon = 10^{-4}$ for complete redistribution with three different Voigt damping parameters
- right: corresponding emergent line profiles and extinction profile shapes. The thermalization depth increases for larger damping because the extended outer wings provide deeper photon escape

TWO-LEVEL SCATTERING WITH BACKGROUND CONTINUUM

RTSA figure 4.13; from Avrett 1965SAOSR.174..101A

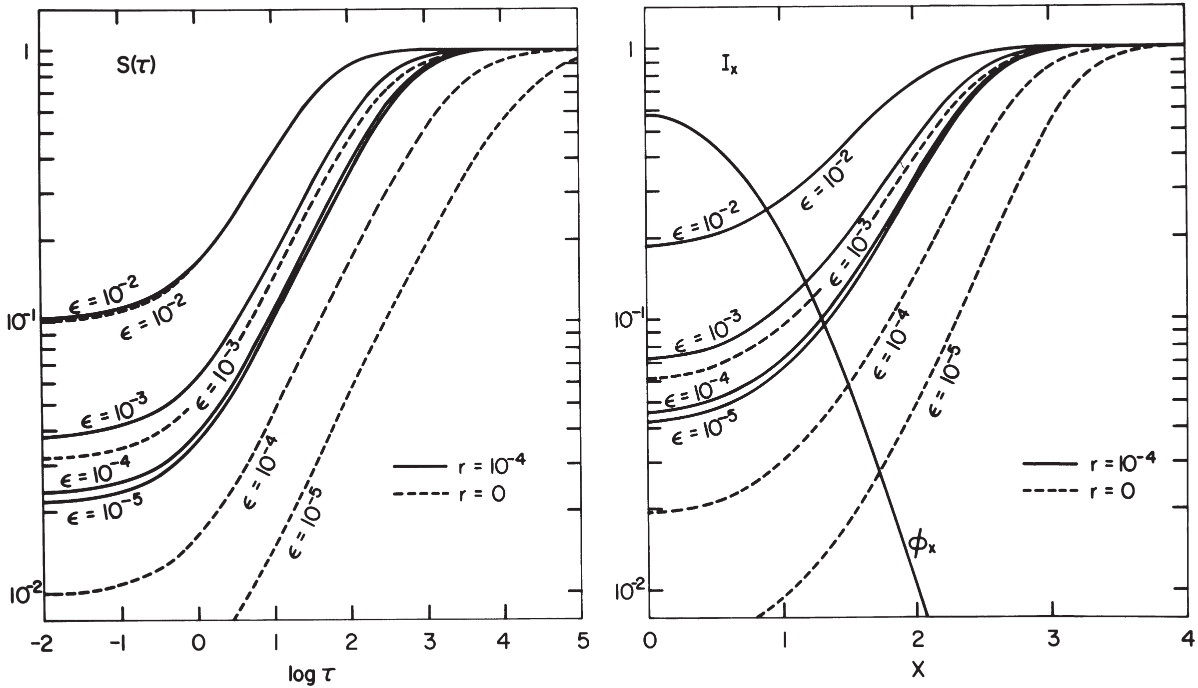
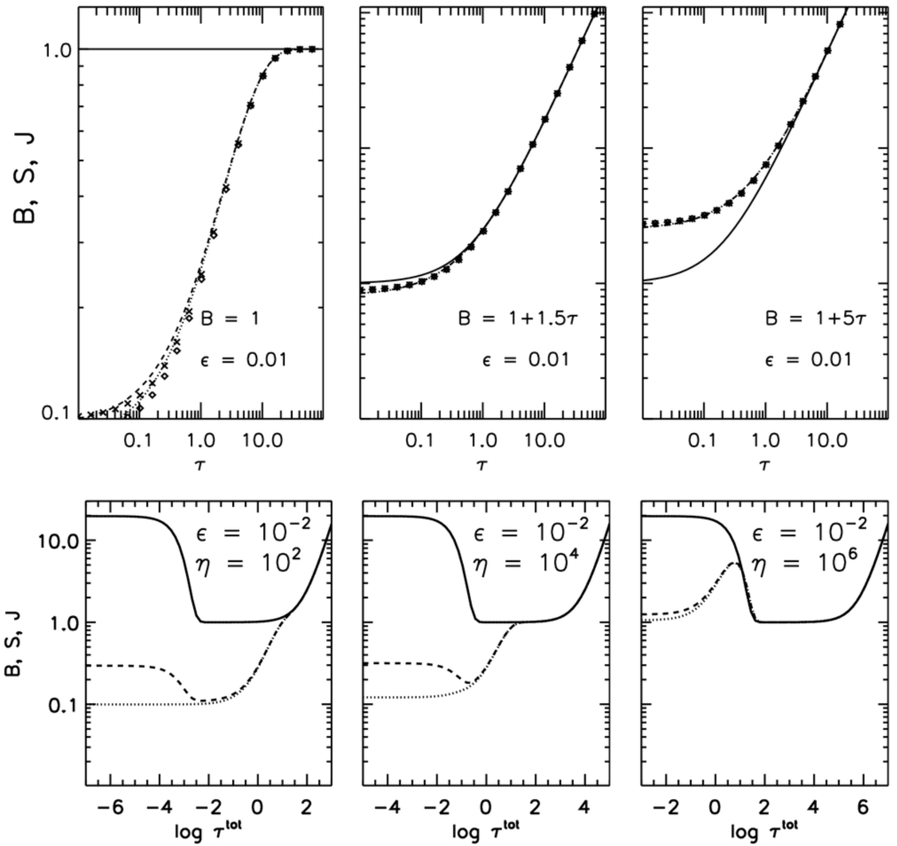


Figure 4.13: Avrett results for two-level-atom lines with complete redistribution and a background continuum. The atmosphere is isothermal. Axis labeling and parameters as for the upper panels of Figure 4.12; the extinction profile $\varphi(x)$ is again Gaussian (righthand panel). *Dashed curves:* $r \equiv \alpha_\nu^c/\alpha_{\nu_0}^l$ set to $r = 0$, describing pure resonance scattering without background continuum. *Solid curves:* $r = 10^{-4}$ or $\eta_{\nu_0} = 10^4$, describing fairly strong lines. Lack of continuum thermalization is unimportant when $r \ll \varepsilon_{\nu_0}$. Lack of collisional destruction is unimportant when $\varepsilon_{\nu_0} \ll r$. From Avrett (1965).

SCATTERING IN EPSILON = 0.01 ATMOSPHERES

$$J_\nu(\tau_\nu) = \mathbf{\Lambda}_{\tau_\nu}[S_\nu(t_\nu)] \qquad S^l_{\nu_0} = (1-\varepsilon_{\nu_0})\, J_{\nu_0} \;+\; \varepsilon_{\nu_0} B_{\nu_0}$$



Krijger 2003rtsa.book.....R

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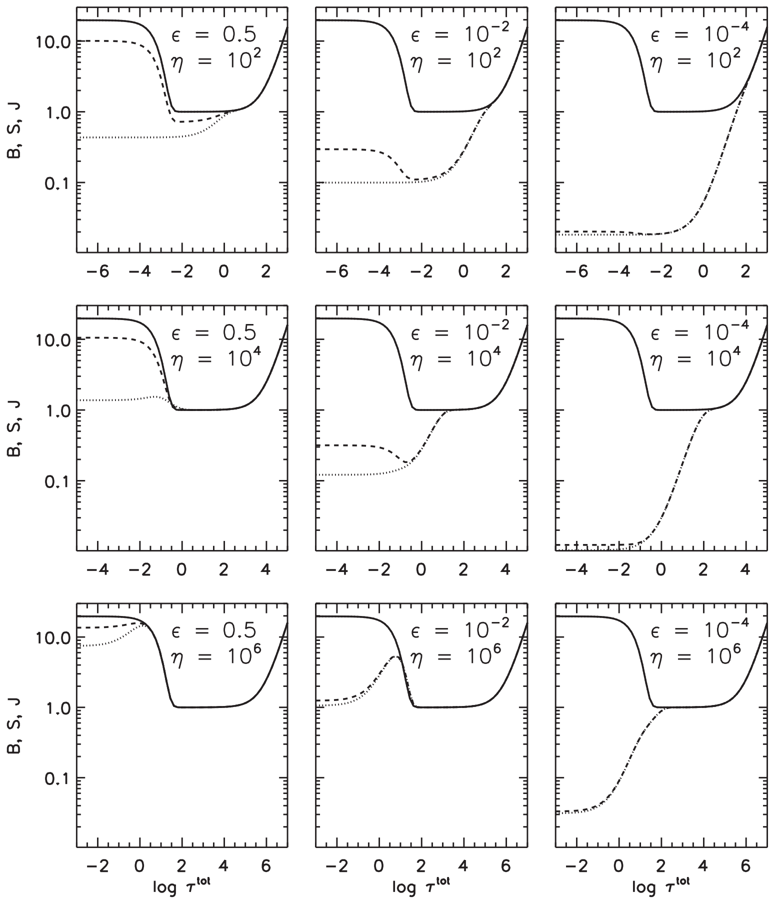
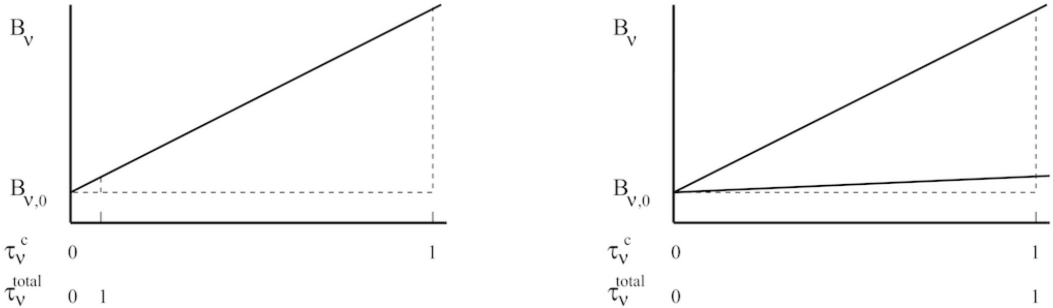
B, J, S FOR SOLAR-LIKE COHERENT LINE SCATTERING

RTSA figure 4.11; Thijs Krijger production



FLAT $S(\tau)$ IN STRONG LINES

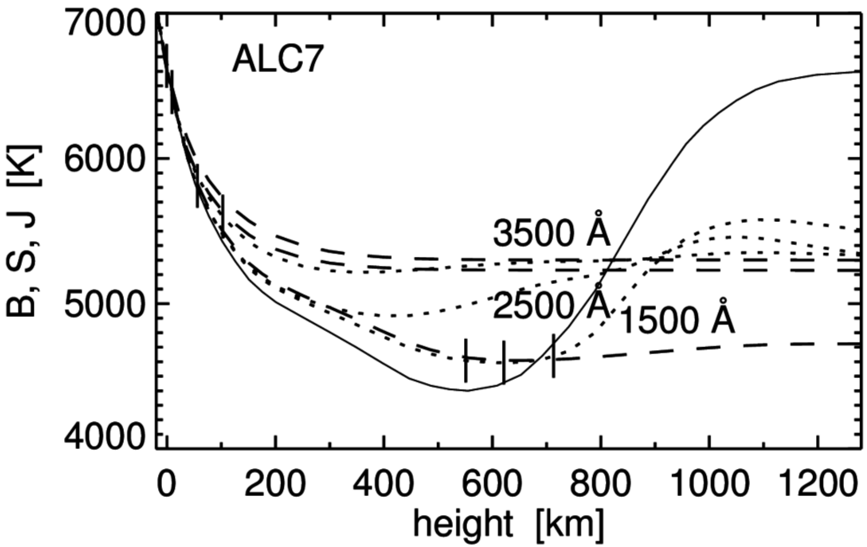
Fig. 4.10 of RTSA



- RE: $B_\nu(\tau_\nu^c) \approx B_{\nu,0}\,(1 + 1.5\,\tau_\nu^c)$ at peak of emergent flux (optical)
- strong line: $\eta_\nu \equiv \alpha_\nu^l / \alpha_\nu^c >> 1$
- tau scaling in line: $\mathrm{d}\tau_\nu^{\mathrm{tot}} = \mathrm{d}\tau_\nu^c + \mathrm{d}\tau_\nu^l = (1 + \eta_\nu)\,\mathrm{d}\tau_\nu^c$
- RE gradient seen by line: $B_\nu(\tau_\nu^{\mathrm{tot}}) \approx B_{\nu,0}\,(1 + 1.5/(1+\eta_\nu)\,\tau_\nu^{\mathrm{tot}})$
- strong lines tend to obey the $\sqrt{\varepsilon}$ law: $S(\tau\!=\!1) << B[T(\tau\!=\!\Lambda)]$

SCATTERING ULTRAVIOLET CONTINUA

Rutten *2016arXiv161105308R*



- photospheric $T(h)$ gradient set in optical by RE
- ultraviolet $B[T(h)]$ much steeper from Wien
- no $B(\tau)$ flattening from strong-line extinction
- Λ operator produces $J > B$

SOLAR SPECTRUM FORMATION: THEORY

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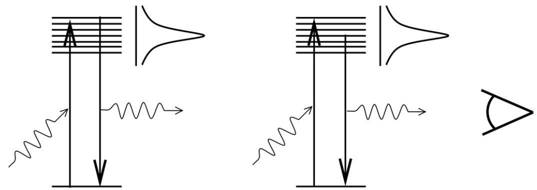
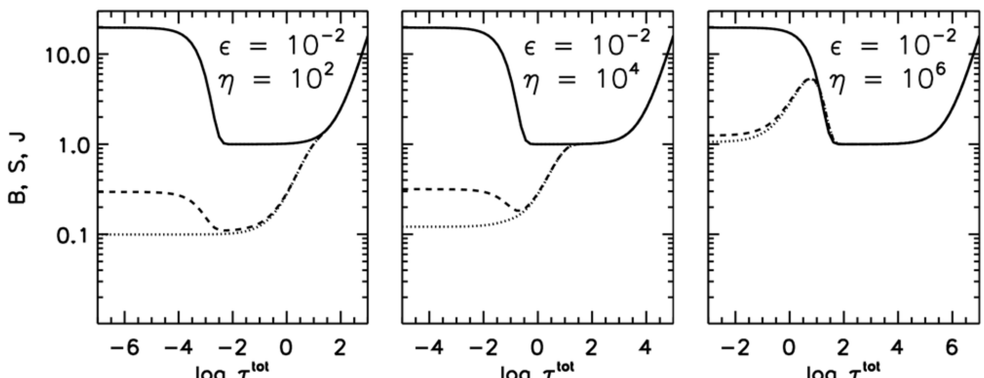
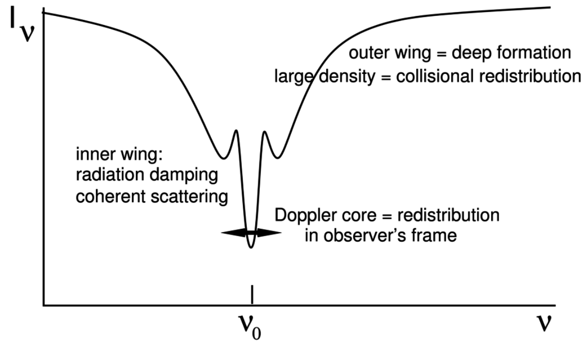
FREQUENCY COHERENCE OR REDISTRIBUTION

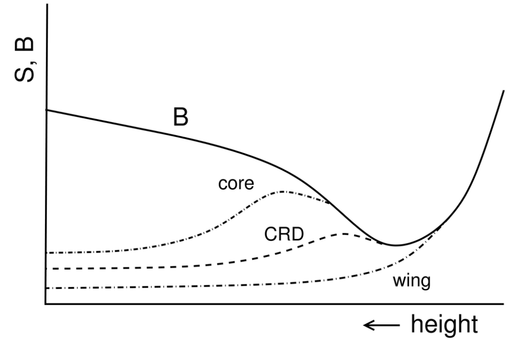


- Eddington: does a re-emitting atom remember at which frequency it was excited?
- yes $=$ coherent scattering: incoming and outgoing photons same frequency
- no $=$ complete redistribution: outgoing takes fresh sample of the probability distribution
- Doppler redistribution: coherent scattering per atom, ensemble Dopplershifts for observer
- collisional redistribution: "reshuffling while atom sits in upper state"
- schematic illustration: coherent scattering in different parts of a strong spectral line



PARTIAL FREQUENCY REDISTRIBUTION PER CARTOON





- Doppler core: monofrequent ("coherent") scatttering per atom (n its moving frame); Doppler redistribution over parcel Doppler width for observer (snag: microturbulence?)
- inner damping wing: Heisenberg ⇒ coherent scattering with Doppler redistribution
- outer damping wing: at large density collisional damping ⇒ complete redistribution
- if the line is so strong that radiation damping dominates in the inner wings (high formation at low collider density) then the inner-wing photons are independent Doppler-wide ensembles with their own line source functions
- inner-wing line source functions decouple deeper from the Planck function than the core source function due to smaller opacity: they represent weaker lines
- the PRD core source function decouples further out than for complete redistribution because core photons cannot escape from deeper layers via occasional wing sampling

PARTIAL REDISTRIBUTION CLASSIC

Hummer *1962MNRAS.125...21H*

NON-COHERENT SCATTERING

I. THE REDISTRIBUTION FUNCTIONS WITH DOPPLER BROADENING

David G. Hummer

(Received 1962 July 12)

Summary

The redistribution in frequency of radiation scattered from moving atoms is examined in some generality, allowing for the different types of scattering which occur in the atom's rest frame under different circumstances. Some general formulae are obtained and a number of explicit results are given. Finally some attention is devoted to the properties of the redistribution functions and to the methods which may be used for computing them.

In this paper we obtain a very general redistribution function for the physically realistic situations in which scattering, according to an arbitrary redistribution function and an arbitrary phase function in the atom's rest frame, is further modified by the Doppler effect. We obtain explicit formulae for the redistribution functions in four cases. They are, with the Roman numeral which will subsequently identify them:

Zero natural line width (I).
Radiation damping with coherence in the atom's rest frame (II).
Radiation and collision damping with complete redistribution in the atom's frame (III).
Resonance scattering (IV).

$$R_{\mathrm{I}}(x, \mathbf{n}; x', \mathbf{n}') = \frac{g(\mathbf{n}, \mathbf{n}')}{4\pi^2 \sin\gamma} \exp\left[-x'^2 - (x - x'\cos\gamma)^2 \csc^2\gamma\right]$$

$$R_{\mathrm{II}}(x, \mathbf{n}; x', \mathbf{n}') = \frac{g(\mathbf{n}, \mathbf{n}')}{4\pi^2 \sin\gamma} \exp\left[-\left(\frac{x-x'}{2}\right)^2 \csc^2\left(\frac{\gamma}{2}\right)\right] H\left(\sigma \sec\frac{\gamma}{2}, \frac{x+x'}{2} \sec\frac{\gamma}{2}\right)$$

$$R_{\mathrm{III}}(x, \mathbf{n}; x', \mathbf{n}') = \frac{g(\mathbf{n}, \mathbf{n}')}{4\pi^2 \sin\gamma} \frac{\sigma}{\pi} \int_{-\infty}^{\infty} \frac{e^{-y^2}}{(x-y)^2 + \sigma^2} H(\sigma \csc\gamma, x' \csc\gamma - y \operatorname{ctn}\gamma)\, dy$$

$$R_{\mathrm{IV}}(x, \mathbf{n}; x', \mathbf{n}') = \frac{g(\mathbf{n}, \mathbf{n}')}{2\pi^2 \sin\gamma} \frac{\sigma_i \sec\gamma/2}{\pi} \times \int_{-\infty}^{\infty} \frac{e^{-y^2} H(\sigma_j \csc\gamma/2, y \operatorname{ctn}\gamma/2 - x \csc\gamma/2)\, dy}{[(x-x')\sec\gamma/2 - 2y]^2 + (\sigma_i \sec\gamma/2)^2}$$

CLEAREST EXPLANATION SOFAR

Jefferies "Spectral line formation" 1968slf..book.....J

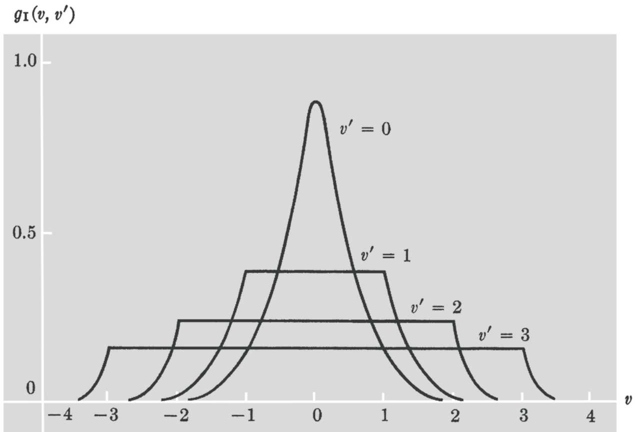


Figure 5.1. The redistribution function $g_{\rm I}(v, v')$ for the case of zero natural line width (see Equations (5.18) and (5.22) of the text); v and v' are respectively the incident and scattered dimensionless frequencies.

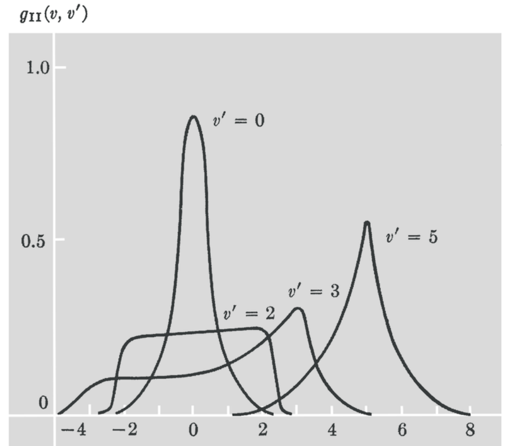


Figure 5.2. The redistribution function $g_{\rm II}(v, v')$ for the case of finite natural line width $a = 10^{-3}$ (see Equations (5.18) and (5.24) of the text); v and v' are respectively the incident and scattered dimensionless frequencies.

In his discussion Hummer distinguishes the two cases of zero and finite natural width a. For the first case, $a = 0$, he finds Unno's (1952*a*) result

$$R_{\rm I}(v, v') = \tfrac{1}{2}\,\mathrm{erfc}\,(|\bar{v}|), \tag{5.22}$$

where the complement to the error function is defined as

$$\mathrm{erfc}\,(x) = \frac{2}{\sqrt{\pi}}\int_x^\infty e^{-t^2}\,dt \tag{5.23}$$

and $|\bar{v}|$ is the larger of $|v|$ and $|v'|$. For a nonzero we have the result, also due to Unno (1952*b*),

$$R_{\rm II}(v, v') = \pi^{-3/2}\int_{(\bar{v}-\underline{v})/2}^{\infty} e^{-u^2}\left[\tan^{-1}\left(\frac{\underline{v}+u}{a}\right) - \tan^{-1}\left(\frac{\bar{v}-u}{a}\right)\right] du, \tag{5.24}$$

where now $\bar{v}$ and $\underline{v}$ are respectively the larger and smaller of v and v'.

The unusual arguments $\bar{v}$ and $\underline{v}$ in these expressions arise because, by assumption, the scattering is coherent in the frame of the atom and only a restricted range of v' is therefore possible for a given absorbed frequency v, and a given atomic velocity. In this respect the problem differs from that of complete redistribution in the atom's frame, for which any frequency v' may be emitted following absorption of a given frequency v.

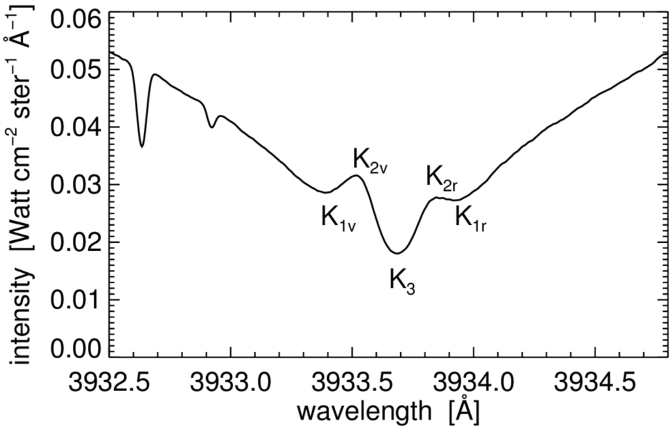
The explanation of these features is straightforward: in the core ($v, v' \lesssim 3$) absorption will be mainly—entirely if $a = 0$—due to those atoms moving with such a velocity as to "see" the photon at their own line center since the atomic absorption coefficient is enormously larger there than at neighboring frequencies. The re-emission at frequency v' is supposed isotropic and so in the rest frame of the atmosphere it is distributed between the frequencies $\pm v$ ($\equiv \pm\Delta\nu/\Delta\nu_{\rm D}$). A closer analysis shows that the distribution is equally probable between these frequencies.

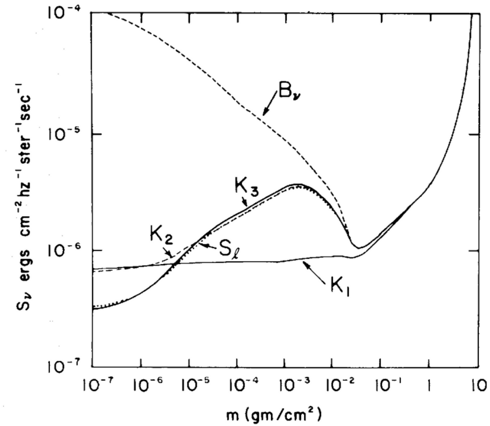
As the incident frequency moves further into the line wings, however, the number of atoms able to absorb in the line center falls rapidly—in fact, like $\exp(-v^2)$. If a were zero, the atom would have no choice but to accept this since in that case its absorption coefficient is zero except at the central frequency. In the practical case where $a \neq 0$ a frequency v_c will be reached in the wings such that beyond v_c the small residual wing absorption coefficient overbalances that due to atoms moving so as to absorb at the line center; in practice, for allowed lines in the visible, v_c is well known to be of order 3. For such wing frequencies, therefore, the predominant absorption will be due to the atoms having small line-of-sight velocities since they are the most numerous. The frequency v' of the re-emitted radiation is therefore more or less equal to that absorbed in this case when the atom itself scatters coherently. In fact, for large v we would expect to find that the probability distribution for v' was centered on v and had a width of the order of one Doppler width.

upshot: Doppler-wide core around observed line center, in far natural-damping wings Doppler-broadened coherency

FORMATION OF Ca II K WITH PARTIAL REDISTRIBUTION

Shine, Milkey, Mihalas *1975ApJ...199..724S*



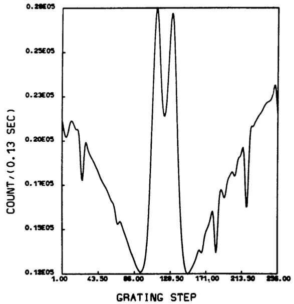


- left: classic naming of Ca II K reversal pattern features (the only non-Gaussian Fraunhofer-line cores!)
- right: in CRD $S^l_{\lambda_0}$ (solid curve) maps the minimum temperature into the Ca II K_1 dips
- PRD S^l_λ departs individually from B_λ for each ensemble of non-redistributed photons
- no such 1D explanation for the $K_{2v} > K_{2r}$ and $K_{1v} > K_{1r}$ asymmetries
- actually, the average profile is dominated by acoustic shocks in internetwork and magnetic concentrations in network
- the Wilson-Bappu effect (stellar core width – luminosity correlation) remains unexplained

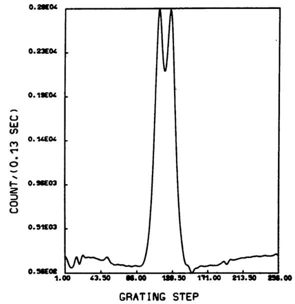
MAJOR PRD LINES

Lemaire et al. 1981A&A...103..160L: observed profiles from plage

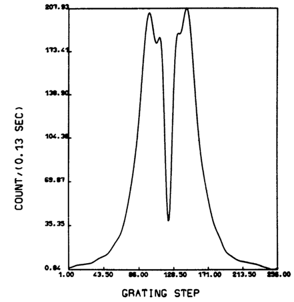
Ca II K



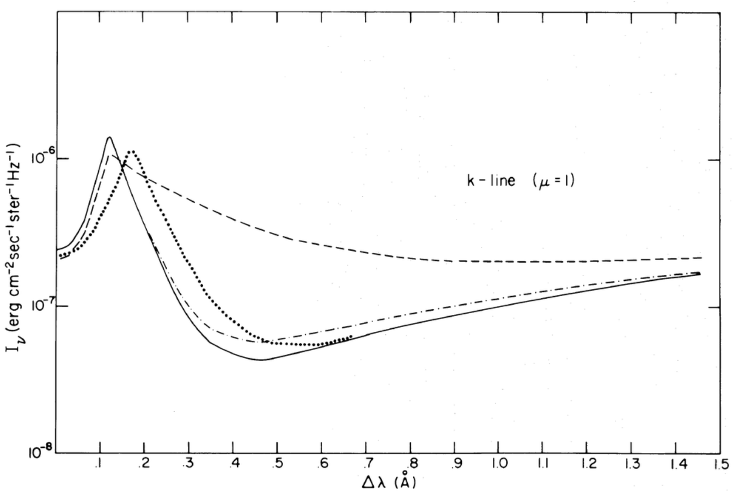
Mg II k



Lyα



Milkey & Mihalas 1974ApJ...192..769M: computed half Mg II k profiles for PRD



RECENT DEVELOPMENTS IN PRD LINE SYNTHESIS

- *RH code: Uitenbroek 2001ApJ...557..389U*
  - Rybicky & Hummer: not $\mathbf{\Lambda}(S)$ but $\mathbf{\Psi}(j)$ iteration; preconditioning
  - overlappping lines
  - 1D, 2D, 3D, spherical versions
- *RH 1.5D: Pereira & Uitenbroek 2015A&A...574A...3P*
  - 1.5D $=$ column-by-column
  - massively parallel
  - also molecular lines (but Kurucz lines in LTE)
- *angle-dependent redistribution: Leenaarts et al. 2012A&A...543A.109L*
  - good summary PRD theory and equations
  - non-stationary atmosphere requires angle-dependent PRD
  - hybrid approximation: transform to gas parcel frame, assume angle-averaged PRD ($\approx$ angle dependent from deep isotropy), transform back
- *towards Bifrost PRD: Sukhorukov & Leenaarts 2017A&A...597A..46S*
  - hybrid approximation for small memory
  - linear frequency interpolation for speed
  - 252$\times$252$\times$496 grid, 1024 CPUs: 2 days for Mg II k $\approx$ doable
- *next: 3D PRD with multigrid (Bjørgen & Leenaarts 2017A&A...599A.118B)*

SOLAR SPECTRUM FORMATION: THEORY

Robert J. Rutten
https://webspace.science.uu.nl/~rutte101

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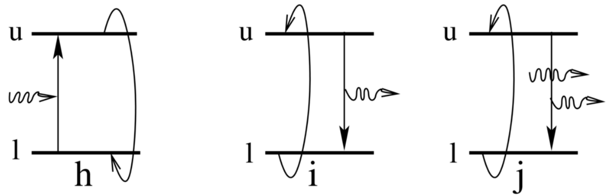
MULTI-LEVEL DETOURS

All "interlocking" paths involving a photon in the beam



Detour source function with detour transition probabilities D_{ul} D_{lu}

$$S^{\mathrm{d}}_{\nu_0} = \frac{2h{\nu_0}^3}{c^2} \frac{1}{\dfrac{g_u}{g_l}\dfrac{D_{ul}}{D_{lu}} - 1} \equiv \frac{2h{\nu_0}^3}{c^2} \frac{1}{\mathrm{e}^{h\nu_0/kT_{\mathrm{d}}} - 1} = B_{\nu_0}(T_{\mathrm{d}})$$

Collision and conversion photon-loss probabilities for sharp-line atoms

$$\varepsilon_{\nu_0} \equiv \frac{\alpha^{\mathrm{a}}_{\nu_0}}{\alpha^{\mathrm{s}}_{\nu_0} + \alpha^{\mathrm{a}}_{\nu_0} + \alpha^{\mathrm{d}}_{\nu_0}} = \frac{C_{ul}\left(1 - \mathrm{e}^{-h\nu_0/kT}\right)}{A_{ul} + C_{ul}\left(1 - \mathrm{e}^{-h\nu_0/kT}\right) + D_{ul}\left(1 - \mathrm{e}^{-h\nu_0/kT_{\mathrm{d}}}\right)}$$

$$\eta_{\nu_0} \equiv \frac{\alpha^{\mathrm{d}}_{\nu_0}}{\alpha^{\mathrm{s}}_{\nu_0} + \alpha^{\mathrm{a}}_{\nu_0} + \alpha^{\mathrm{d}}_{\nu_0}} = \frac{D_{ul}\left(1 - \mathrm{e}^{-h\nu_0/kT_{\mathrm{d}}}\right)}{A_{ul} + C_{ul}\left(1 - \mathrm{e}^{-h\nu_0/kT}\right) + D_{ul}\left(1 - \mathrm{e}^{-h\nu_0/kT_{\mathrm{d}}}\right)}$$

Line source function

$$S^{l}_{\nu_0} = (1 - \varepsilon_{\nu_0} - \eta_{\nu_0})\, J_{\nu_0} \;+\; \varepsilon_{\nu_0} B_{\nu_0}(T) \;+\; \eta_{\nu_0} B_{\nu_0}(T_{\mathrm{d}})$$

ALTERNATE NOTATION IN THE (CLASSICAL) LITERATURE

E.g., Jefferies "Spectral Line Formation" 1968slf..book.....J

Normalized photon destruction and photon conversion

$$\varepsilon'_{\nu_0} \equiv \frac{\alpha^{\mathrm{a}}_{\nu_0}}{\alpha^{\mathrm{s}}_{\nu_0}} = \frac{\varepsilon_{\nu_0}}{1-\varepsilon_{\nu_0}-\eta_{\nu_0}} = \frac{C_{ul}\left(1-\mathrm{e}^{-h\nu_0/kT}\right)}{A_{ul}}$$

$$\eta'_{\nu_0} \equiv \frac{\alpha^{\mathrm{d}}_{\nu_0}}{\alpha^{\mathrm{s}}_{\nu_0}} = \frac{\eta_{\nu_0}}{1-\varepsilon_{\nu_0}-\eta_{\nu_0}} = \frac{D_{ul}\left(1-\mathrm{e}^{-h\nu_0/kT_{\mathrm{d}}}\right)}{A_{ul}}$$

Extinction coefficient

$$\alpha^{l}_{\nu_0} = \alpha^{\mathrm{s}}_{\nu_0}\,(1+\varepsilon'_{\nu_0}+\eta'_{\nu_0})$$

Line source function

$$S^{l}_{\nu_0} = \frac{J_{\nu_0} + \varepsilon'_{\nu_0} B_{\nu_0}(T) + \eta'_{\nu_0} B_{\nu_0}(T_{\mathrm{d}})}{1+\varepsilon'_{\nu_0}+\eta'_{\nu_0}}$$

Complete redistribution

$$S^{l}_{\nu_0} = (1-\varepsilon_{\nu_0}-\eta_{\nu_0})\,\overline{J}^{\varphi}_{\nu_0} + \varepsilon_{\nu_0} B_{\nu_0}(T) + \eta_{\nu_0} B_{\nu_0}(T_{\mathrm{d}}) = \frac{\overline{J}^{\varphi}_{\nu_0} + \varepsilon'_{\nu_0} B_{\nu_0}(T) + \eta'_{\nu_0} B_{\nu_0}(T_{\mathrm{d}})}{1+\varepsilon'_{\nu_0}+\eta'_{\nu_0}}$$

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RADIATIVE COOLING

RTSA 7.3.2

Radiative equilibrium condition

$$\begin{aligned}\Phi_{\rm tot}(z) &\equiv \frac{{\rm d}\mathcal{F}_{\rm rad}(z)}{{\rm d}z} = 0 \\ &= 4\pi \int_0^\infty \alpha_\nu(z)\,[S_\nu(z) - J_\nu(z)]\;{\rm d}\nu \\ &= 2\pi \int_0^\infty \int_{-1}^{+1} [j_{\nu\mu}(z) - \alpha_{\nu\mu}(z)\,I_{\nu\mu}(z)]\;{\rm d}\mu\,{\rm d}\nu\end{aligned}$$

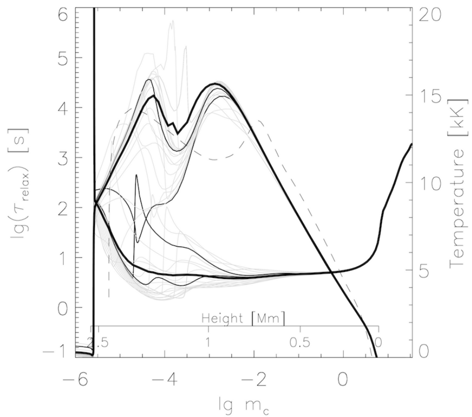
Net radiative cooling in a two-level atom gas

$$\begin{aligned}\Phi_{ul} &= 4\pi\alpha^l_{\nu_0}(S^l_{\nu_0} - \overline{J}_{\nu_0}) \\ &= 4\pi j^l_{\nu_0} - 4\pi\alpha^l_{\nu_0}\overline{J}_{\nu_0} \\ &= h\nu_0\left[n_u(A_{ul} + B_{ul}\overline{J}_{\nu_0}) - n_l B_{lu}\overline{J}_{\nu_0}\right] \\ &= h\nu_0\,[n_u R_{ul} - n_l R_{lu}]\end{aligned}$$

Net radiative cooling in a one-level-plus-continuum gas

$$\Phi_{ci} = 4\pi\, n_i^{\rm LTE} b_c \int_{\nu_0}^\infty \sigma_{ic}(\nu)\left[B_\nu\left(1 - \,{\rm e}^{-h\nu/kT}\right) - \frac{b_i}{b_c}J_\nu\left(1 - \frac{b_c}{b_i}\,{\rm e}^{-h\nu/kT}\right)\right]\;{\rm d}\nu$$

DETAILED BALANCING



Hydrogen ionization/recombination relaxation timescale throughout the solar-like shocked Radyn atmosphere. The timescale for settling to equilibrium at the local temperature is very long, 15–150 min, in the chromosphere but much shorter, only seconds, in shocks in which hydrogen partially ionizes.
Carlsson & Stein 2002ApJ...572..626C

net radiative and collisional downward rates (Wien approximation)

$$n_u R_{ul} - n_l R_{lu} \approx \frac{4\pi}{h\nu_0}\, n_l^{\rm LTE}\, b_u\, \sigma^l_{\nu_0} \left(B_{\nu_0} - \frac{b_l}{b_u} \overline{J}_{\nu_0} \right)$$ zero for $S = \overline{J}$, no heating/cooling

$$n_u C_{ul} - n_l C_{lu} = n_l C_{lu} \left(\frac{b_u}{b_l} - 1 \right) = b_u n_l^{\rm LTE} C_{lu} \left(1 - \frac{b_l}{b_u} \right)$$ zero for $b_u = b_l$, LTE S^l

dipole approximation for atom collisions with electrons (Van Regemorter 1962)

$$C_{ul} \approx 2.16 \left(\frac{E_{ul}}{kT} \right)^{-1.68} T^{-3/2} \frac{g_l}{g_u} N_{\rm e}\, f$$

Einstein relation

$$C_{lu} = C_{ul} \frac{g_l}{g_u} \, {\rm e}^{-E_{ul}/kT}$$

C_{ul} is not very temperature sensitive (any collider will do); C_{lu} has Boltzmann sensitivity

LAMBDA ITERATION

Lambda operator

$$J_\nu(\tau_\nu) = \mathbf{\Lambda}_\nu[S_\nu(t)]$$

Two-level coherent scattering

$$S^l_\nu(\tau_\nu) = (1 - \varepsilon_\nu(\tau_\nu))\,\mathbf{\Lambda}_\nu[S^l_\nu(t_\nu)] + \varepsilon_\nu(\tau_\nu)B_\nu(\tau_\nu)$$

Drop indices

$$S = (1 - \varepsilon)\,\mathbf{\Lambda}_\nu[S] + \varepsilon B$$

$$S = (1 - (1 - \varepsilon)\,\mathbf{\Lambda})^{-1}\,[\varepsilon B]$$

Iteration instead of inversion

$$S^{(n+1)} = (1 - \varepsilon)\,\mathbf{\Lambda}[S^{(n)}] + \varepsilon B$$

Convergence

$$S^{(n+1)} - S^{(n)} = (1 - \varepsilon)\,\mathbf{\Lambda}_\nu[S^{(n)}] + \varepsilon B - S^{(n)}$$

Large τ, small ε

$$S^{(n+1)} - S^{(n)} \approx \mathbf{\Lambda}_\nu[S^{(n)}] - S^{(n)} \approx S^{(n)} - S^{(n)} \approx 0$$

ACCELERATED LAMBDA ITERATION

Operator splitting (Cannon): define $\mathbf{\Lambda}^*$ as a valid but fast approximation

$$\mathbf{\Lambda}_\nu = \mathbf{\Lambda}^* + (\mathbf{\Lambda}_\nu - \mathbf{\Lambda}^*)$$

Still exact

$$J_\nu = \mathbf{\Lambda}^*_\nu[S] + (\mathbf{\Lambda}_\nu - \mathbf{\Lambda}^*_\nu)[S]$$

Iteration inserting $n+1$ also on the righthand side

$$S^{(n+1)} = (1-\varepsilon)\,\mathbf{\Lambda}^*[S^{(n+1)}] + (1-\varepsilon)(\mathbf{\Lambda}_\nu - \mathbf{\Lambda}^*)[S^{(n)}] + \varepsilon B$$

Reshuffle

$$S^{(n+1)} - (1-\varepsilon)\,\mathbf{\Lambda}^*[S^{(n+1)}] = (1-\varepsilon)\,\mathbf{\Lambda}_\nu[S^{(n)}] + \varepsilon B - (1-\varepsilon)\,\mathbf{\Lambda}^*[S^{(n)}] = S^{\mathrm{FS}} - (1-\varepsilon)\,\mathbf{\Lambda}^*[S^{(n)}]$$

Inversion of only the approximate operator (FS = formal solution)

$$S^{(n+1)} = (1-(1-\varepsilon)\,\mathbf{\Lambda}^*)^{-1}\left[S^{\mathrm{FS}} - (1-\varepsilon)\,\mathbf{\Lambda}^*[S^{(n)}]\right]$$

Convergence

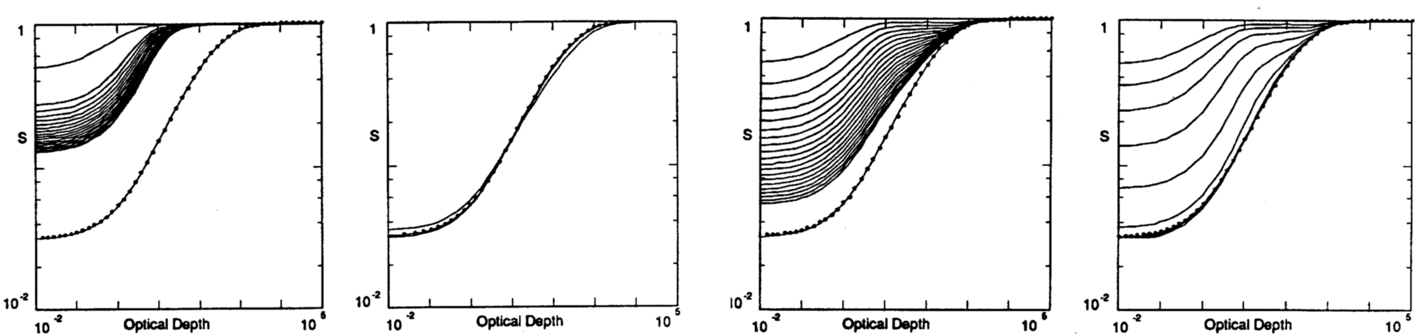
$$S^{(n+1)} - S^{(n)} = (1-(1-\varepsilon)\,\mathbf{\Lambda}^*)^{-1}\left[S^{\mathrm{FS}} - S^{(n)}\right]$$

Acceleration

$$(1-(1-\varepsilon)\,\mathbf{\Lambda}^*)^{-1} \approx 1/\varepsilon$$

LAMBDA ITERATION EXAMPLES

Auer 1991sabc.conf....9A (Crivellari, Hummer, Hubený)



- isothermal semi-infinite atmosphere
- constant $\varepsilon_{\nu_0} = 10^{-3}$, complete redistribution, Gaussian profile
- display = 20 successive S^l estimates + correct S^l

- A: classical $\mathbf{\Lambda}$ iteration
- B: ALI with Scharmer operator (local Eddington-Barbier along LOS)
- C: ALI with the diagonal of the $\mathbf{\Lambda}$ matrix
- D: idem with conjugate vector acceleration

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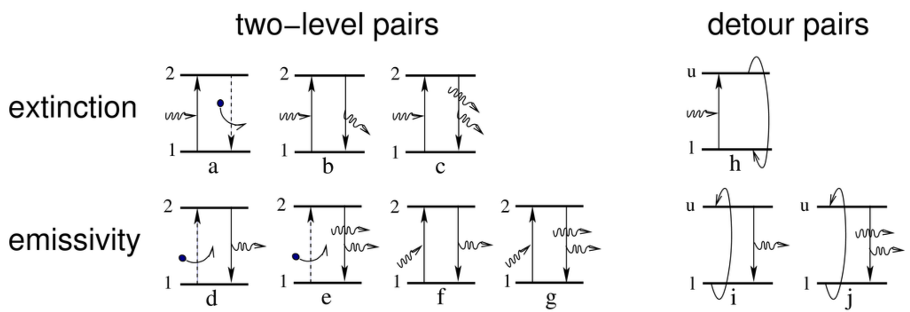
LINE FORMATION AS SEEN BY THE ATOM



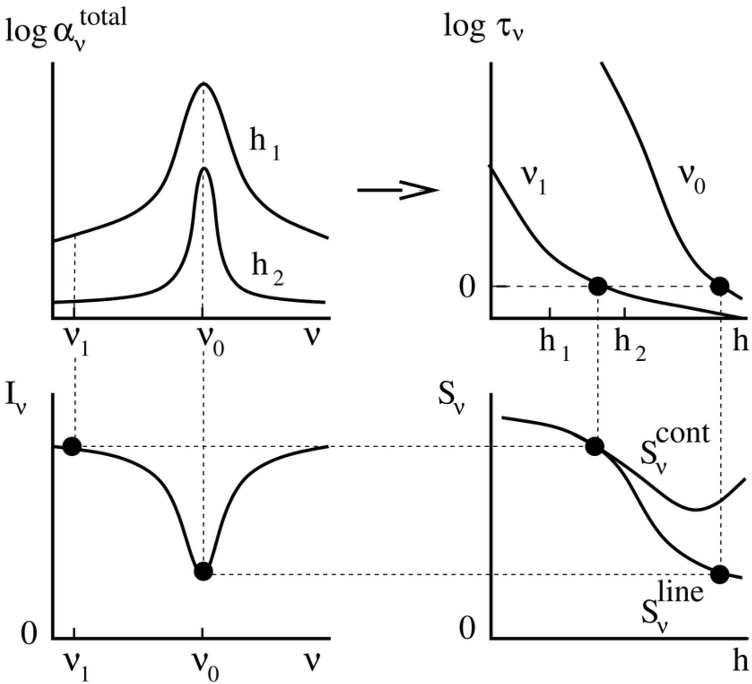
- *pair combinations*
  - beam of interest to the right
  - a / d + e = collisional destruction / creation of beam photons
  - b + h / f + i + j scattering & detour photons out / into beam (c, g cancel)

- *equilibria*
  - LTE: a + d + e dominate; bb Boltzmann $f(T)$, bf Saha $f(T,N_\mathrm{e})$
  - CE: d only; bb $f(T,N_\mathrm{e})$, bf $f(T)$
  - NLTE, NSE: scattering and/or detours important; bb and bf $f(T, N_e, , \overline{J}_{ul}, \overline{J}_{ij}, \overline{J}_{ic})\,[t]$

- *line extinction and line source function*
  - $\alpha^l = \alpha^\mathrm{a} + \alpha^\mathrm{s} + \alpha^\mathrm{d}$ absorption + scattering + detour extinction
  - $\varepsilon \equiv \alpha^\mathrm{a}/\alpha^l$ destruction probability $\quad$ $\eta \equiv \alpha^\mathrm{d}/\alpha^l$ detour probability
  - $S^l = (1-\varepsilon-\eta)\,\overline{J} \;+\; \varepsilon B(T) \;+\; \eta S^d$ $\quad$ $\overline{J}$: mean mean intensity $\quad$ S^d: all detours

REALISTIC SOLAR ABSORPTION LINE

- extinction: bb peak in $\eta_\nu \equiv \alpha_l/\alpha_c$ becomes lower and narrower at larger height
- optical depth: $\tau_\nu \equiv -\int \alpha_\nu^{\rm total}\,{\rm d}h$ increases nearly log-linearly with geometrical depth
- source function: split for line (bb) and continuous (bf, ff, electron scattering) processes
- intensity: Eddington-Barbier for $S_\nu^{\rm total} = (\alpha_c S_c + \alpha_l S_l)/(\alpha_c + \alpha_l) = (S_C + \eta_\nu S_l)/(1+\eta_\nu)$



BASIC RADIATIVE TRANSFER EQUATIONS

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specific intensity	$I_\nu(\vec{r}, \vec{l}, t)$ erg cm^{-2} s^{-1} Hz^{-1} ster^{-1}
emissivity	j_ν erg cm^{-3} s^{-1} Hz^{-1} ster^{-1}
extinction coefficient	α_ν cm^{-1} σ_ν cm^2 part^{-1} κ_ν cm^2 g^{-1}
source function	$S_\nu = \sum j_\nu / \sum \alpha_\nu$
radial optical depth	$\tau_\nu(z_0) = \int_{z_0}^{\infty} \alpha_\nu \, \mathrm{d}z$
plane-parallel transport	$\mu \, \mathrm{d}I_\nu / \mathrm{d}\tau_\nu = I_\nu - S_\nu$
thin cloud	$I_\nu = I_0 + (S_\nu - I_0)\, \tau_\nu$
thick emergent intensity	$I_\nu^+(0, \mu) = \int_0^\infty S_\nu(\tau_\nu)\, \mathrm{e}^{-\tau_\nu/\mu} \, \mathrm{d}\tau_\nu / \mu$
Eddington-Barbier	$I_\nu^+(0, \mu) \approx S_\nu(\tau_\nu = \mu)$
mean mean intensity	$\overline{J}_{\nu_0}^{\varphi} = \frac{1}{2} \int_0^\infty \int_{-1}^{+1} I_\nu \, \varphi(\nu - \nu_0) \, \mathrm{d}\mu \, \mathrm{d}\nu$
photon destruction	$\varepsilon_\nu = \alpha_\nu^{\mathrm{a}} / (\alpha_\nu^{\mathrm{a}} + \alpha_\nu^{\mathrm{s}}) \approx C_{ul} / (A_{ul} + C_{ul})$
complete redistribution	$S_{\nu_0}^l = (1 - \varepsilon_{\nu_0})\, \overline{J}_{\nu_0}^{\varphi} + \varepsilon_{\nu_0} B_{\nu_0}$
isothermal atmosphere	$S_{\nu_0}(0) = \sqrt{\varepsilon_{\nu_0}}\, B_{\nu_0}$

KEY LINE FORMATION EQUATIONS

population departure coefficients

$$b_l = n_l / n_l^{\rm LTE} \qquad\qquad b_u = n_u / n_u^{\rm LTE}$$

Zwaan: $n^{\rm LTE} =$ Saha-Boltzmann fraction of $N_{\rm el}$ Harvard: n/n_c (main stages $\approx 1/b_c$)

general line extinction and line source function

$$\alpha_\lambda^l = \frac{\pi e^2}{m_{\rm e} c} \frac{\lambda^2}{c} b_l \frac{n_l^{\rm LTE}}{N_{\rm el}} N_{\rm H} A_{\rm el} f_{lu} \varphi \left[1 - \frac{b_u}{b_l} \frac{\chi}{\varphi} \, {\rm e}^{-hc/\lambda kT} \right] \qquad S_\lambda^l = \frac{2hc^2}{\lambda^5} \frac{\psi/\varphi}{\frac{b_l}{b_u} \, {\rm e}^{hc/\lambda kT} - \frac{\chi}{\varphi}}$$

CRD approximation: $\psi = \chi = \varphi$ Wien approximation: neglect stimulated parts

$$\alpha^l \approx b_l \; \alpha^{\rm LTE} \qquad\qquad S^l \approx (b_u/b_l) \; B(T)$$

PRD: Lyα, Mg II h & k, Ca II H & K, strong UV Wien: up to Hα ($\lambda T = hc/k$ at 21 900 K)

probabilities per extinction of collisional photon destruction and of detour photon conversion

$$\varepsilon \equiv \frac{\alpha^{\rm a}}{\alpha^{\rm s} + \alpha^{\rm a} + \alpha^{\rm d}} \qquad\qquad \eta \equiv \frac{\alpha^{\rm d}}{\alpha^{\rm s} + \alpha^{\rm a} + \alpha^{\rm d}}$$

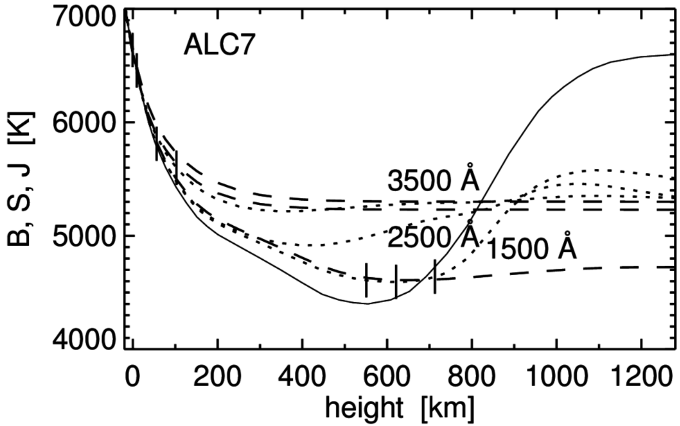
line source function (for CRD, monofrequent for PRD)

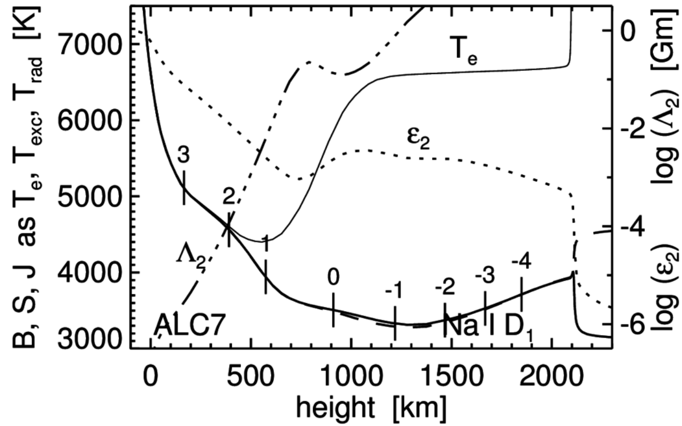
$$S^l = (1 - \varepsilon - \eta) \, \overline{J} \; + \; \varepsilon B(T) \; + \; \eta S^{\rm d}$$

"source" $=$ local addition of new photons into beam per local extinction in terms of energy

$\overline{J} \equiv (1/4\pi) \iint I \varphi \, {\rm d}\Omega \, {\rm d}\lambda$ reservoir $\varepsilon \, B$ thermal creation $\eta \, S^{\rm d}$ detour production

ULTRAVIOLET b-f SCATTERING VERSUS OPTICAL b-b SCATTERING

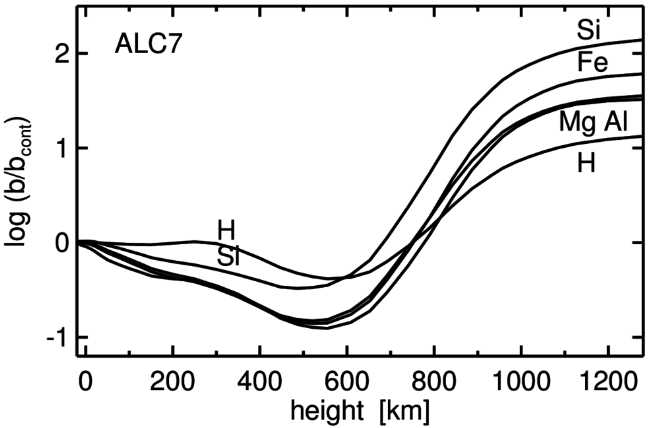


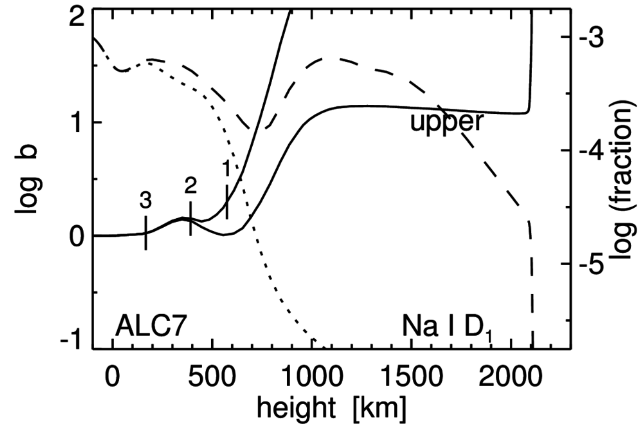


- *scattering ultraviolet continua*
  - scatter outward from deep photosphere
  - $B_\lambda(\tau_\lambda)$ steeper than defining optical RE gradient $B_{5000}(\tau_{5000}) \sim 1 + 1.5\,\tau_{5000}$
  - source function follows J, not steep drop in B

- *scattering optical line (Na I D$_1$)*
  - scatters outward from upper photosphere
  - optical depth scale compressed compared to $\tau_{5000} \Rightarrow B_\lambda(\tau_\lambda) \approx$ flat $\sim$ "isothermal"
  - source function doesn't care that temperature rises again

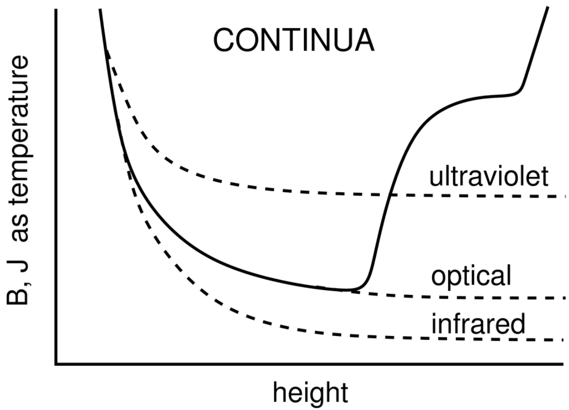
ULTRAVIOLET b-f SCATTERING VERSUS OPTICAL b-b SCATTERING

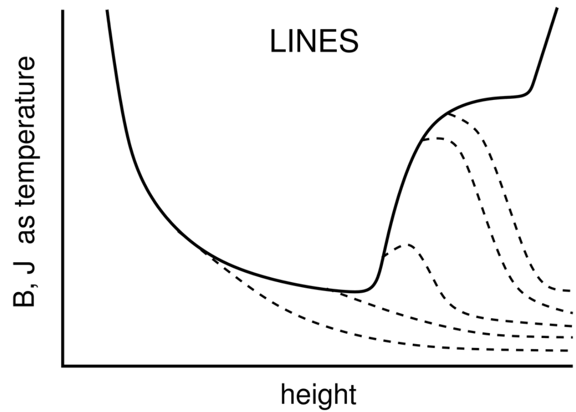




- *scattering ultraviolet continua*
  - $J - S$ translates into standard dip + rise pattern
  - photospheric minority-species lines have $b_{\rm cont}\!\approx\!1$ and b_1 extinction depletion
  - H I: Balmer continuum has same pattern in $b_2/b_{\rm cont}$ (H I top $\sim$ neutral metal)
- *scattering optical line (Na I D$_1$)*
  - no photospheric dip because alkalis suffer photon suction: photon-loss replenishment from population reservoir in low-lying continuum
  - steep b_l increase from ultraviolet overionization $\Rightarrow B_\lambda(\tau_\lambda) \approx$ flat $\sim$ "isothermal"
  - b_u/b_l split characteristic for scattering lines

SUMMARY 1D SCATTERING SOURCE FUNCTIONS





- *continua*
  - optical: $J \approx B$ for radiative equilibrium
  - ultraviolet: $S \approx J > B \rightarrow$ overionization of minority neutrals
  - infrared: $J < B$ but J doesn't matter since $H^-_{\rm ff}$ and $H_{\rm ff}$ have $S = B$
- *lines*
  - $\mathrm{d}B/\mathrm{d}\tau = \mathrm{d}B/\mathrm{d}(\tau^c + \tau^l)$ much less steep, so closer to isothermal $S \approx \sqrt{\varepsilon}\, B$
  - for stronger lines S sees more of the model chromosphere
  - PRD lines have frequency-dependent core-to-wing $S \approx J$ curves like these

SOLAR SPECTRUM FORMATION: THEORY

Robert J. Rutten
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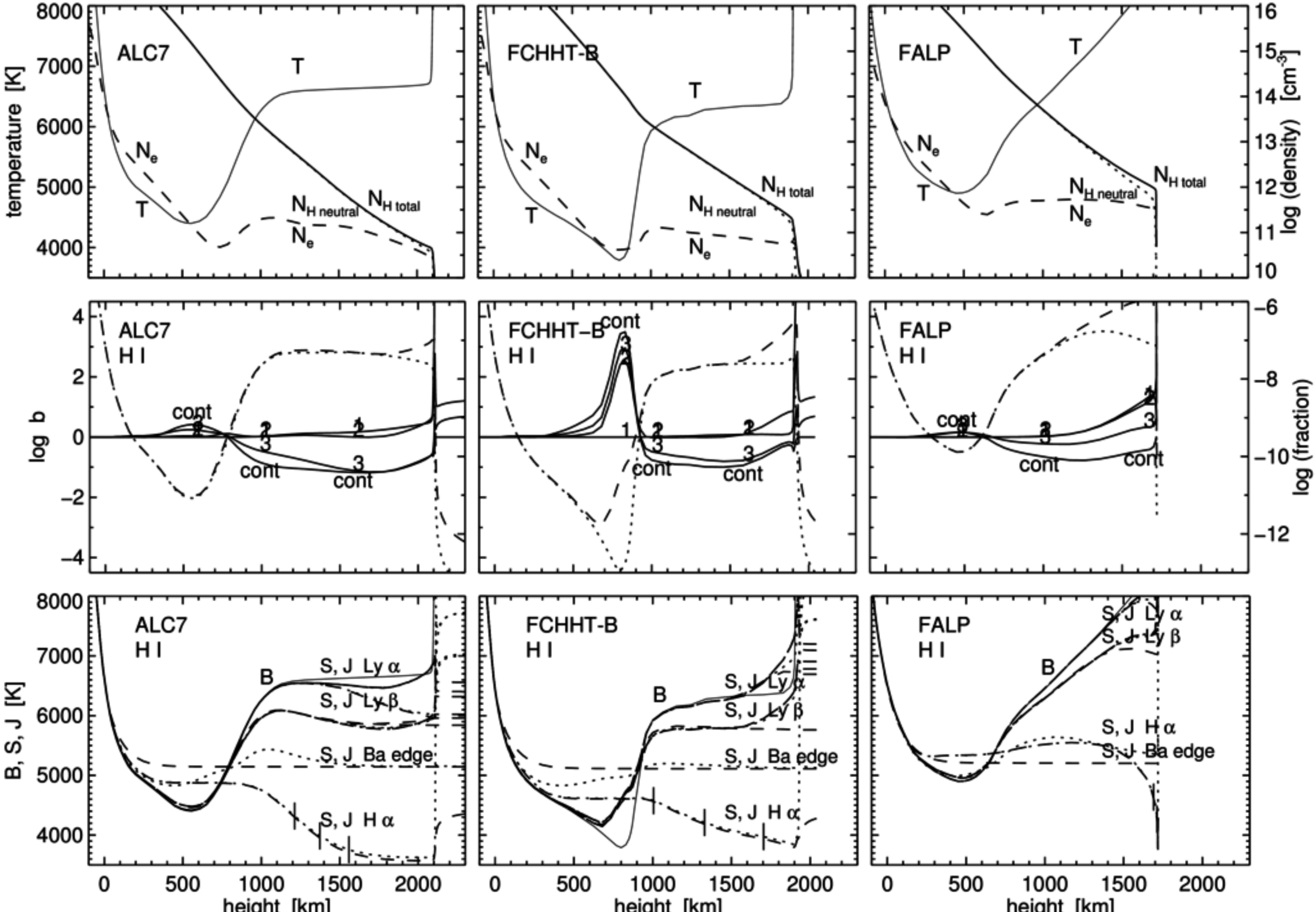
EXPLAIN EVERYTHING – INCLUDING SIMILARITIES AND DIFFERENCES

ALC7: *2008ApJS..175..229A* *FCHHT-B:* *2009ApJ...707..482F* *FALP:* *1993ApJ...406..319F*



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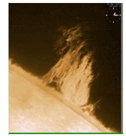
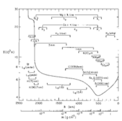
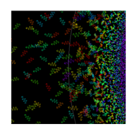
WHO WANTS TO KNOW WHAT WHAT FOR?



- *optically thin cloud modeler* courtesy Tom Berger
  - $I_\nu(\tau_\nu) = I_\nu(0)\,\mathrm{e}^{-\tau_\nu} + \int_0^{\tau_\nu} S_\nu(t_\nu)\,\mathrm{e}^{-(\tau_\nu - t_\nu)}\,\mathrm{d}t_\nu \approx I_\nu(0)\,\mathrm{e}^{-\tau_\nu(D)} + S_\nu\left(1 - \mathrm{e}^{-\tau_\nu(D)}\right)$
  - off limb: $I_\nu(0) = 0$ but how do I solve confusion?
  - on disk: how do I define the unseen $I_\nu(0)$?

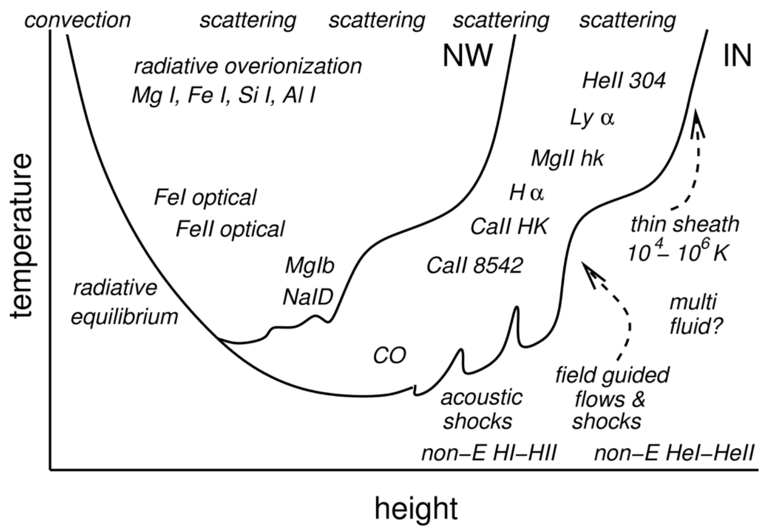


- *optically thick Eddington-Barbier inverter* courtesy Gene Avrett
  - $I_\nu^+(\tau_\nu\!=\!0, \mu) = \int_0^\infty S_\nu(t_\nu)\,\mathrm{e}^{-t_\nu/\mu}\,\mathrm{d}t_\nu/\mu \approx S_\nu(\tau_\nu = \mu)$
  - can I get away with $\tau_\nu = \tau_\nu^{\mathrm{LTE}}$ and $S_\nu = B_\nu$?
  - at what height does my line form and how does it tell me $T, N_\mathrm{e}, \vec{v}, \vec{B}$?



- *excitable atom in the solar atmosphere* courtesy Mats Carlsson
  - what colliders and photons are available for my excitation?
  - shall I emit or extinct a photon in the observer's direction?
  - do I muck with coherency?

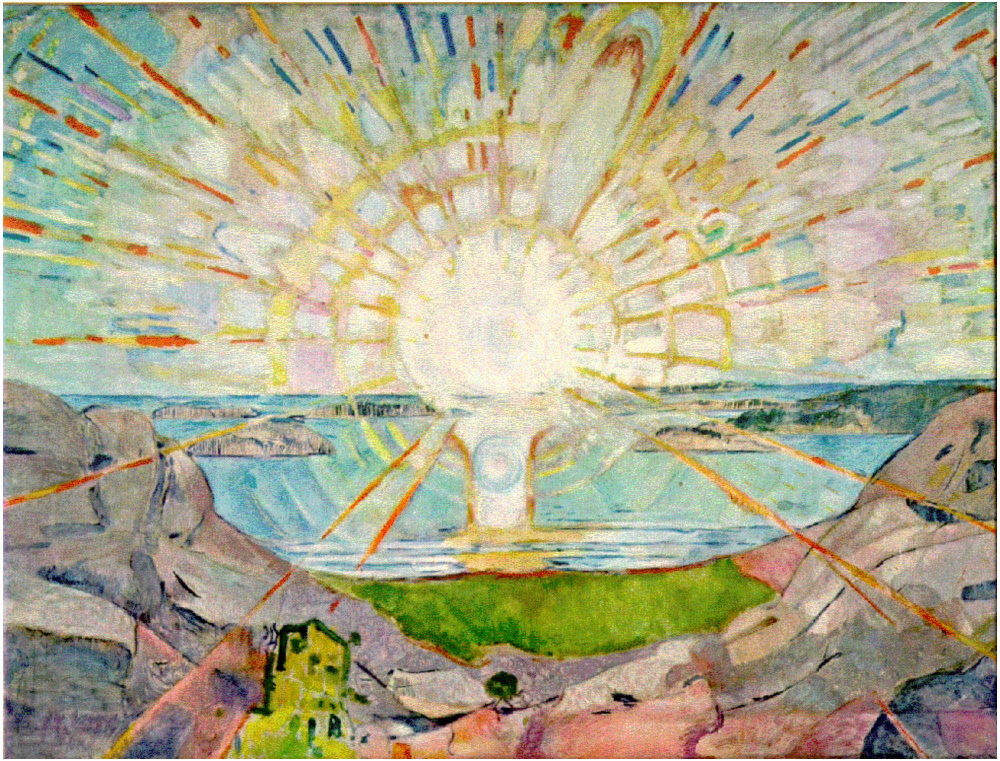
NLTE SCENE



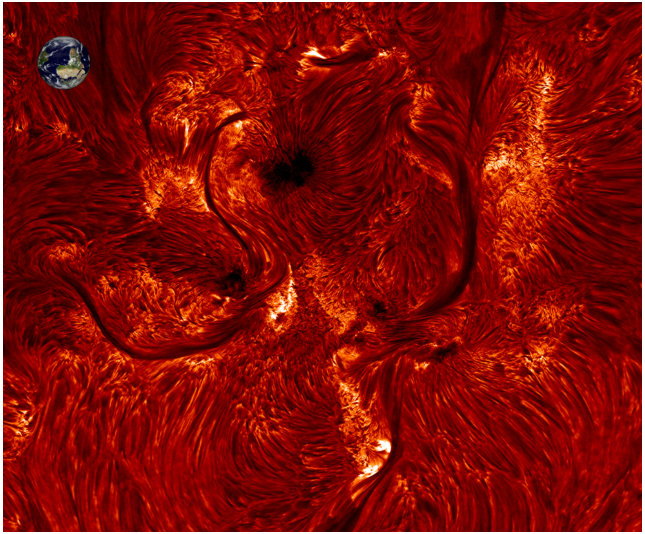
- message: scattering – scattering – scattering – scattering
- UV continua: $S \approx J$, minority overionization (deep) and underionization (high)
- photospheric lines: opacity $<$ LTE (Fe I) or source function $>$ LTE (Fe II)
- chromospheric lines: $S \approx J$, NEQ(t) opacities (H, He), PRD (Lyα, h & k, H & K)
- p.m. NLTE funnies:
  interlocking (Ce II in H & K, Canfield 1971A&A....10...64C)
  pumping (Fe II in H & K, Cram et al. 1980ApJ...241..374C)
  replenishment (Mg I 12 μm, Carlsson et al. 1992A&A...253..567C)
  suction (Na I & K I, Bruls et al. 1992A&A...265..237B)

Edvard Munch (1863–1944)

"The camera cannot compete with the brush and the palette so long as it cannot be used in heaven or hell"



BEAUTY IN SHARPNESS



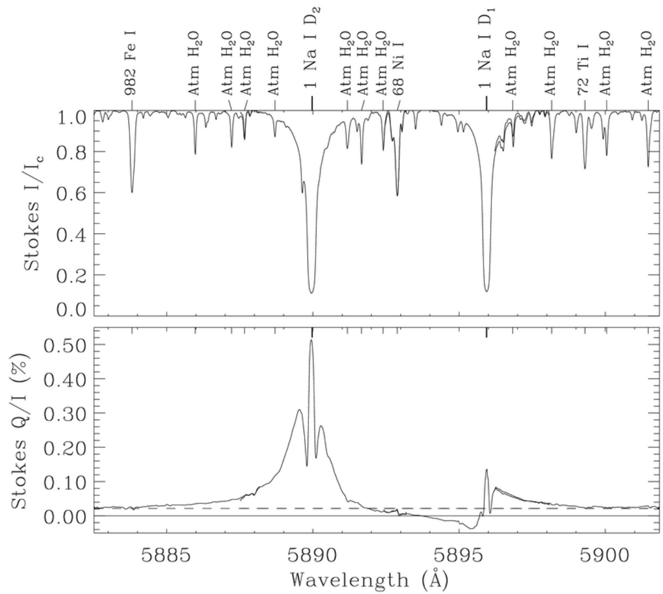
DOT picture of the sun. Solar active region AR10786 in a mosaic of 2 by 2 DOT images taken on June 8, 2005. The field measures 182 x 133 arcsec. The inserted Earth photograph indicates the scale. The sunspot umbrae remain dark in Halpha. Many so-called fibrils emanate away from the sunspots. They outline magnetic connections between different areas, like iron filings around a bar magnet. The many fibrils show how complex solar magnetism is arranged within the solar atmosphere. The whitish areas surrounding the sunspots are plage, where large numbers of magnetic elements cluster together. The long slender dark structures are active region filaments. They end in bipolar regions where both positive and negative magnetic fields emerge through the solar surface. The coloring is artificial.

https://webspace.science.uu.nl/~rutte101/dot

ON BEAUTY AND COMFORT

"It is a great human comfort to look at a distant star and to realize that the light that reaches our eyes contains the Na D lines, the same sodium lines that produce our yellow street lighting, and that we understand exactly how these lines are formed. The sodium atoms in that faraway star obey physical laws that we know and understand in great detail. Isn't that wonderful!"

Steven Weinberg, TV interview (2000)



"The polarization peaks in the line cores, in particular that of the D_1 line, remain enigmatic"

Stenflo, Keller, Gandorfer 2000A&A...355..789S

Stefan Dreizler: Lecture on stellar atmospheres

Stellar Atmospheres: Radiative Equilibrium

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start: dawn of astrophysics exercises literature 101-intro

basics: basic quantities flux intensity conservation exam constant S_ν plane-atmosphere RT EB CF+RF formation cartoons E-B exam $\Lambda(S)$

LTE 1D static: Planck EB-line-limb continuous opacity electron donors Saha-Boltzmann line broadening LTE line equations

NLTE descriptions: solar radiation processes bb equilibria Einstein coefficients line source function formal temperatures departure coefficients lasering population + transport equations

scattering: 2-level atoms sharp atom CZ demo scattering equations results

refinements: partial redistribution multi-level detours radiative cooling balancing Λ iteration

course summary: all bb pairs NLTE line cartoon equation summary key equations scattering cont & line NLTE summary cartoon homework

course finish: H I exam moral conclusion

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LITERATURE

- *Mihalas*
  - "Stellar Atmospheres" (1970) (?)
  - "Stellar Atmospheres" (1978) (*)
  - with Hubený: "Theory of Stellar Atmospheres" (2014)
- *simpler*
  - Novotny: "Introduction to stellar atmospheres and interiors" (1973)
  - Gray: "The observation and analysis of stellar photospheres" (2005)
  - Böhm-Vitense: "Introduction to stellar astrophysics I, II" (1989)
- *harder*
  - Cannon: "The transfer of spectral line radiation" (1985)
  - Mihalas & Mihalas: "Foundations of Radiation Hydrodynamics" (1984) (*)
  - Castor: "Radiation Hydrodynamics" (2004)
- *my stuff*
  - IART: bachelors-level radiative transfer (1991, 2015, 20??) (*)
  - RTSA: masters-level Mihalas popularization (2003, 20??) (*)
  - ISSF: bachelors-level introduction to NLTE (1993) (*)
  - Monterey: PhD-level refresher NLTE chromospheric lines for IRIS (2012) (*)
  - Cartagena: tutorial non-E hydrogen diagnostics for ALMA (2016) (*)

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SOLAR SPECTRUM FORMATION: THEORY

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course finish: H I exam moral conclusion

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BASIC QUANTITIES

Monochromatic emissivity

$$dE_\nu \equiv j_\nu \, dV \, dt \, d\nu \, d\Omega \qquad dI_\nu(s) = j_\nu(s)\, ds$$

units j_ν: erg cm^{-3} s^{-1} Hz^{-1} ster^{-1} $\quad I_\nu$: erg cm^{-2} s^{-1} Hz^{-1} ster^{-1}

Monochromatic extinction coefficient

$$dI_\nu \equiv -\sigma_\nu n \, I_\nu \, ds \qquad dI_\nu \equiv -\alpha_\nu \, I_\nu \, ds \qquad dI_\nu \equiv -\kappa_\nu \rho \, I_\nu \, ds$$

units: cm^2 per particle (physics) cm^2 per cm^3 = per cm (RTSA) cm^2 per gram (astronomy)

Monochromatic source function

$$S_\nu \equiv j_\nu/\alpha_\nu = j_\nu/\kappa_\nu\rho \qquad S_\nu^{\rm tot} = \frac{\sum j_\nu}{\sum \alpha_\nu} \qquad S_\nu^{\rm tot} = \frac{j_\nu^c + j_\nu^l}{\alpha_\nu^c + \alpha_\nu^l} = \frac{S_\nu^c + \eta_\nu S_\nu^l}{1+\eta_\nu} \qquad \eta_\nu \equiv \alpha_\nu^l/\alpha_\nu^c$$

thick: (α_ν, S_ν) more independent than (α_ν, j_ν) stimulated emission negatively into α_ν, κ_ν

Transport equation with τ_ν as optical thickness along the beam

$$\frac{dI_\nu}{ds} = j_\nu - \alpha_\nu I_\nu \qquad \frac{dI_\nu}{\alpha_\nu \, ds} = S_\nu - I_\nu \qquad d\tau_\nu \equiv \alpha_\nu ds \qquad \tau_\nu(D) = \int_0^D \alpha_\nu ds \qquad \frac{dI_\nu}{d\tau_\nu} = S_\nu - I_\nu$$

Plane-parallel transport equation with τ_ν as radial optical depth and μ as viewing angle

$$d\tau_\nu \equiv -\alpha_\nu \, dz \qquad \tau_\nu(z_0) = -\int_\infty^{z_0} \alpha_\nu \, dz \qquad \mu \equiv \cos\theta \qquad \mu \frac{dI_\nu}{d\tau_\nu} = I_\nu - S_\nu$$

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FLUX

Monochromatic flux [erg s^{-1} cm^{-2} Hz^{-1}] or [W m^{-2} Hz^{-1}] (solid angle)

$$\mathcal{F}_\nu(\vec r, \vec n, t) \equiv \int I_\nu \cos\theta \, d\Omega = \int_0^{2\pi}\!\!\int_0^\pi I_\nu \cos\theta \sin\theta \, d\theta \, d\varphi$$

Ingoing and outgoing

$$\mathcal{F}_\nu(z) = \int_0^{2\pi}\!\!\int_0^{\pi/2} I_\nu \cos\theta \sin\theta \, d\theta \, d\varphi + \int_0^{2\pi}\!\!\int_{\pi/2}^{\pi} I_\nu \cos\theta \sin\theta \, d\theta \, d\varphi \equiv \mathcal{F}_\nu^+(z) - \mathcal{F}_\nu^-(z)$$

Axial symmetry (plane-parallel layers)

$$\mathcal{F}_\nu(z) = 2\pi \int_0^\pi I_\nu \cos\theta \sin\theta \, d\theta = 2\pi \int_0^1 \mu I_\nu \, d\mu - 2\pi \int_0^{-1} \mu I_\nu \, d\mu = \mathcal{F}_\nu^+(z) - \mathcal{F}_\nu^-(z)$$

Surface flux of a non-irradiated spherical star

$$\mathcal{F}_\nu^{\rm surface} \equiv \mathcal{F}_\nu^+(r=R) = \pi \overline{I_\nu^+} \qquad \overline{I_\nu^+} = \text{average over apparent stellar disk from faraway}$$

"Astrophysical flux"

$$\pi F_\nu \equiv \mathcal{F}_\nu \quad \text{so that} \quad F_\nu = \overline{I_\nu^+}$$

"Eddington flux"

$$H_\nu(z) \equiv \frac{1}{2}\int_{-1}^{+1} \mu I_\nu \, d\mu$$

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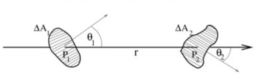
CONSERVATION OF INTENSITY



- consider all photons that travel first through area A_1 around P_1 and then also through area A_2 around P_2
- in P_1: $\Delta E_1 \equiv I_1 \cos\theta_1 \, \Delta A_1 \, \Delta t \, \Delta\nu \, \Delta\Omega_1$. This proportionality holds in the infinitesimal limit $\Delta \to d$
- likewise in P_2: $\Delta E_2 \equiv I_2 \cos\theta_2 \, \Delta A_2 \, \Delta t \, \Delta\nu \, \Delta\Omega_2$
- insert the solid angle that each area extends on the sky of the other: $\Delta\Omega_1 = \cos\theta_2 \, \Delta A_2/r^2$ and $\Delta\Omega_2 = \cos\theta_1 \, \Delta A_1/r^2$
  $\Delta E_1 \equiv I_1 \cos\theta_1 \, \Delta A_1 \, \Delta t \, \Delta\nu \, \cos\theta_2 \, \Delta A_2/r^2 = I_1 \cos\theta_1 \cos\theta_2 \, \Delta A_1 \, \Delta A_2 \, \Delta t \, \Delta\nu / r^2$
  $\Delta E_2 \equiv I_2 \cos\theta_2 \, \Delta A_2 \, \Delta t \, \Delta\nu \, \cos\theta_1 \, \Delta A_1/r^2 = I_2 \cos\theta_1 \cos\theta_2 \, \Delta A_1 \, \Delta A_2 \, \Delta t \, \Delta\nu / r^2$
- since $\Delta E_1 = \Delta E_2$ (the given stream of photons) it follows that $I_1 = I_2$

This macroscopic conservancy for the propagation of light in vacuum expresses the photon property of non-decay (mass zero).

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EXAM: INTENSITY CONSERVATION ALONG A BEAM

- You can use a magnifying glass to start a fire with sunshine. What is the intensity in its focus? Why does it heat?
- The 4-m DKIST will have four times the aperture size of the 1-m SST. Compare the exposure times needed for solar observation.
  Christoph Keller's answer
- A supergiant telescope resolves granules on a solar-analog star 10 lightyears away. What exposure is needed compared to DKIST?
- An amateur astronomer in Iceland photographs an Apollo landing site on the Moon through her 25-cm telescope at 100 times magnification with a Canon camera. Compare the required exposure time to when she uses her Canon with its standard lens in the Holuhraun.
  My answer
- Why are the largest solar telescopes smaller than the largest night-time telescopes?

CONSTANT SOURCE FUNCTION

Transport equation along the beam (τ_ν = optical thickness)

$$\frac{\mathrm{d}I_\nu}{\mathrm{d}\tau_\nu} = S_\nu - I_\nu \qquad I_\nu(\tau_\nu) = I_\nu(0)\,\mathrm{e}^{-\tau_\nu} + \int_0^{\tau_\nu} S_\nu(t_\nu)\,\mathrm{e}^{-(\tau_\nu - t_\nu)}\,\mathrm{d}t_\nu$$

Invariant S_ν

$$I_\nu(D) = I_\nu(0)\,\mathrm{e}^{-\tau_\nu(D)} + S_\nu\left(1 - \mathrm{e}^{-\tau_\nu(D)}\right)$$

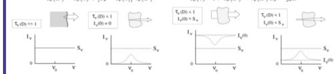
example: $S_\nu = B_\nu$ for all continuum and line processes in an isothermal cloud

Thick object

$$I_\nu(D) \approx S_\nu$$

Thin object

$$I_\nu(D) \approx I_\nu(0) + [S_\nu - I_\nu(0)]\,\tau_\nu(D) \qquad I_\nu(0)=0:\ I_\nu(D) \approx \tau_\nu(D)\,S_\nu = j_\nu D$$



RADIATIVE TRANSFER IN A PLANE ATMOSPHERE

Radial optical depth

$$\mathrm{d}\tau_\nu = -\kappa_\nu \rho\,\mathrm{d}r$$

r radial   Hubený $\tau_{\nu\mu}$ κ_ν cm^2/gram α_ν cm^{-1} = cm^2/cm^3 σ_ν cm^2/particle

Transport equation

$$\mu\,\frac{\mathrm{d}I_\nu}{\mathrm{d}\tau_\nu} = I_\nu - S_\nu$$

Integral form

$$I_\nu^-(\tau_\nu,\mu) = -\int_0^{\tau_\nu} S_\nu(t_\nu)\,\mathrm{e}^{-(t_\nu-\tau_\nu)/\mu}\,\mathrm{d}t_\nu/\mu \qquad I_\nu^+(\tau_\nu,\mu) = +\int_{\tau_\nu}^{\infty} S_\nu(t_\nu)\,\mathrm{e}^{-(t_\nu-\tau_\nu)/\mu}\,\mathrm{d}t_\nu/\mu$$

"formal solution"   NB: both directions   pm: Doppler anisotropy S_ν

Emergent intensity without irradiation

$$I_\nu(0,\mu) = (1/\mu)\int_0^\infty S_\nu(\tau_\nu)\ \mathrm{e}^{-\tau_\nu/\mu}\,\mathrm{d}\tau_\nu$$

Eddington–Barbier approximation

$$\boxed{I_\nu(0,\mu) \approx S_\nu(\tau_\nu=\mu)}$$

exact for linear $S_\nu(\tau_\nu)$

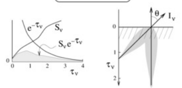
EDDINGTON-BARBIER APPROXIMATION

Emergent intensity without irradiation

$$I_\nu(0,\mu) = (1/\mu)\int_0^\infty S_\nu(\tau_\nu)\ \mathrm{e}^{-\tau_\nu/\mu}\,\mathrm{d}\tau_\nu$$

Eddington-Barbier (Milne–Unsöld?) approximation

$$\boxed{I_\nu(0,\mu) \approx S_\nu(\tau_\nu=\mu)}$$



- wrong: *"the radiation comes from $\tau_\nu=1$" or "the photons escape at $\tau_\nu=1$"*
- correct: *"the emergent radiation is characterized by the source function at $\tau_\nu=1$"*
- beware: a spectral line may instead be formed in a "cloud" at any height
- unresolved star: $F_\nu(0) \approx S(\tau_\nu=2/3)$ with $F_\nu(0) = 2\int_0^1 \mu I_\nu(0)\,\mathrm{d}\mu = \overline{I_\nu(0)}$

CONTRIBUTION & RESPONSE FUNCTIONS

Eddington-Barbier approximation

$$I_\nu(0,\mu) \approx S_\nu(\tau_\nu=\mu)$$

Intensity contribution function

$$I_\nu = \int S_\nu\ \mathrm{e}^{-\tau_\nu}\,\mathrm{d}\tau_\nu \qquad \mathrm{CF}_I \equiv \frac{\mathrm{d}I_\nu}{\mathrm{d}h} = S_\nu\ \mathrm{e}^{-\tau_\nu}\,\frac{\mathrm{d}\tau_\nu}{\mathrm{d}h} = j_\nu\ \mathrm{e}^{-\tau_\nu}$$

Line depression contribution function (Magain 1986A&A...163..135M)

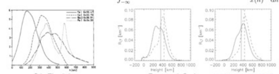
$$D \equiv \frac{I_\nu^C - I_\nu}{I_\nu^C} = \int S_D\,\mathrm{e}^{-\tau_D}\,\mathrm{d}\tau_D \qquad S_D = \frac{1 - S_l/I_C}{1 + (\kappa_C/\kappa_l)\,S_C/I_C} \qquad \kappa_D = \kappa_l + \kappa_C(S_C/I_C)$$

Intensity response function

$$I_\nu = \int_{-\infty}^{+\infty} R_{I,X}(\nu,h)\,X(h)\,\mathrm{d}h \qquad \Delta I_\nu = \int_{-\infty}^{+\infty} R_{I,X}(\nu,h)\,\Delta X(h)\,\mathrm{d}h$$

Numerical intensity response function (Fossum & Carlsson 2005ApJ...625..556F)

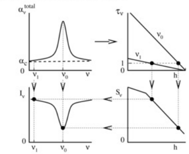
$$\Delta X(h') = x(h')\,H(h'-h) \qquad \Delta I_\nu^h = \int_{-\infty}^{h} R_{I,X}(\nu,h')\,x(h')\,\mathrm{d}h' \qquad R_{I,X}(\nu,h) = \frac{1}{x(h)}\,\frac{\mathrm{d}\Delta I_\nu^h}{\mathrm{d}h}$$



Stix Fig. 6.9   Fossum & Carlsson 2005ApJ...625..556F

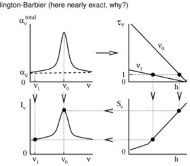
SIMPLE ABSORPTION LINE

- extinction: bb process gives peak in $\alpha_{\rm total} = \alpha_c + \alpha_l = (1+\eta_\nu)\,\alpha_c$
- optical depth: assume height-invariant $\alpha_{\rm total}$ ⇒ linear $(1+\eta_\nu)\,\tau_c$
- source function: assume same for line (bb) and continuous (bf, ff) processes
- use Eddington-Barbier (here nearly exact, why?)



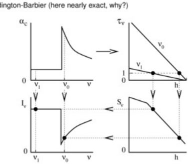
SIMPLE EMISSION LINE

- extinction: bb process gives peak in $\alpha_{\rm total} = \alpha_c + \alpha_l = (1+\eta_\nu)\,\alpha_c$
- optical depth: assume height-invariant $\alpha_{\rm total}$ ⇒ linear $(1+\eta_\nu)\,\tau_c$
- source function: assume same for line (bb) and continuous (bf, ff) processes
- use Eddington-Barbier (here nearly exact, why?)



SIMPLE ABSORPTION EDGE

- extinction: bf process gives edge in α_ν^c, with $\alpha_\nu^c \propto \nu^3$ if hydrogenic
- optical depth: assume height-invariant (unrealistic, why?)
- source function: assume same for the whole frequency range (unrealistic, why?)
- use Eddington-Barbier (here nearly exact, why?)



SIMPLE EMISSION EDGE

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- optical depth: assume height-invariant (unrealistic, why?)
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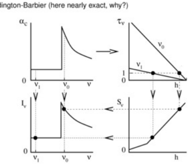
- use Eddington-Barbier (here nearly exact, why?)



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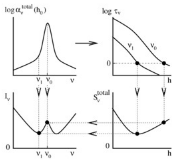
SELF-REVERSED ABSORPTION LINE

- extinction: bb peak with height-dependent amplitude and shape
- optical depth: non-linear even in the log
- source function: decrease followed by increase (any idea why?)
- use Eddington-Barbier (questionable, why?)



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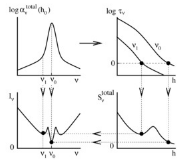
DOUBLY REVERSED ABSORPTION LINE

- extinction: bb peak with height-dependent amplitude and shape
- optical depth: non-linear even in the log
- source function: decrease followed by increase followed by decrease (any idea why?)
- use Eddington-Barbier (questionable, why?)



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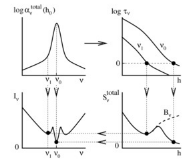
DOUBLY REVERSED ABSORPTION LINE

- extinction: bb peak with height-dependent amplitude and shape
- optical depth: non-linear even in the log
- source function: decrease followed by increase followed by decrease (NLTE scattering)
- use Eddington-Barbier (questionable, why?)



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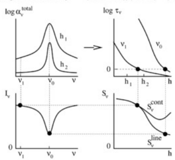
REALISTIC SOLAR ABSORPTION LINE

- extinction: bb peak in $\eta_\nu \equiv \alpha_l/\alpha_c$ becomes lower and narrower at larger height
- optical depth: $\tau_\nu \equiv -\int \alpha_\nu^{total}\,dh$ increases nearly log-linearly with geometrical depth
- source function: split for line (bb) and continuous (bf, ff, electron scattering) processes
- intensity: Eddington-Barbier for $S_\nu^{total} = (\alpha_c S_c + \alpha_l S_l)/(\alpha_c + \alpha_l) = (S_C + \eta_\nu S_l)/(1+\eta_\nu)$



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SOLAR SPECTRUM FORMATION: THEORY

Robert J. Rutten

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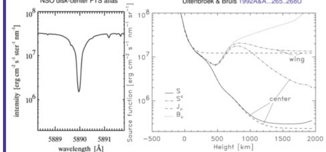
SOLAR NaI D2

NSO disk-center FTS atlas

Uitenbroek & Bruls 1992A&A...265..268U



Na I D_2 is a good example of two-level scattering with complete redistribution: very dark

Eddington-Barbier approximation: line-center $\tau = 1$ at $h \approx 600$ km
chromospheric velocity response but photospheric brightness response

What is the formation height of the blend line in the blue wing?

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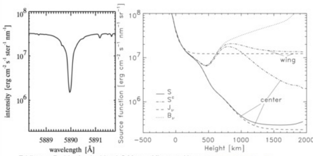
SOLAR NaI D2



Eddington-Barbier for the blends? Moore, Minnaert, Houtgast 1966sst..book.....M:

5888.703		10.	2.	ATM H2O	R4	302	26
5889.637		14.	2.	ATM H2O	R4	401	26
5889.756	•	752.	4.	ATM H2O	R3	401	26
5889.973M	•	752.	120.SS	NA 1(D2)	0.00	1	
5890.203	•	752.	3.	ATM H2O	R4	302	26
5890.495		5.	1.S*	FE 1P	5.06	1313	
5891.178		17.	3.S	ATM H2O	R3	401	17,26
5891.178		17.	3.S	FE 1P	4.65	1179	

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NLTE descriptions: solar radiation processes bb equilibria Einstein coefficients line source function formal temperatures departure coefficients lasering population + transport equations

scattering: 2-level atoms sharp atom CZ demo scattering equations results

refinements: partial redistribution multi-level detours radiative cooling balancing Λ iteration

course summary: all bb pairs NLTE line cartoon equation summary key equations scattering cont & line NLTE summary cartoon homework

course finish: H I exam moral conclusion

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EXPONENTIAL INTEGRALS

Definition with $\mu \equiv 1/w$ to make them an integrals over angle

$$E_n(x) \equiv \int_1^\infty \frac{e^{-xw}}{w^n}\,dw = \int_0^1 e^{-x/\mu}\,\mu^{n-1}\,\frac{d\mu}{\mu}$$

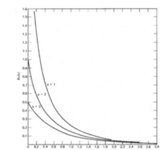

Figure 4.1: The first three exponential integrals $E_n(x)$. $E_1(x)$ has a singularity at $x = 0$. For large x all $E_n(x)$ have $E_n(x) \approx \exp(-x)/x$. From Gray (1992).

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LAMBDA OPERATOR

Exponential integrals

$$E_n(x) \equiv \int_1^\infty \frac{e^{-xw}}{w^n}\,dw = \int_0^1 e^{-x/\mu}\,\mu^{n-1}\,\frac{d\mu}{\mu}$$

Schwarzschild equation

$$J_\nu(\tau_\nu) \equiv \frac{1}{2}\int_{-1}^{+1} I_\nu(\tau_\nu,\mu)\,d\mu = \frac{1}{2}\int_{\tau_\nu}^{\infty} S_\nu(t_\nu)\,E_1(t_\nu-\tau_\nu)\,dt_\nu + \frac{1}{2}\int_0^{\tau_\nu} S_\nu(t_\nu)\,E_1(\tau_\nu-t_\nu)\,dt_\nu$$
$$= \frac{1}{2}\int_0^\infty S_\nu(t_\nu)\,E_1(|t_\nu-\tau_\nu|)\,dt_\nu$$

Lambda operator

$$\Lambda_\tau[f(t)] \equiv \frac{1}{2}\int_0^\infty f(t)\,E_1(|t-\tau|)\,dt$$

Schwarzschild equation

$$J_\nu(\tau_\nu) = \frac{1}{2}\int_0^\infty S_\nu(t_\nu)\,E_1(|t_\nu-\tau_\nu|)\,dt_\nu = \Lambda_{\tau_\nu}[S_\nu(t_\nu)]$$

Generalized lambda operators (Scharmer, Hubený)

$\overline{J}_\nu(\tau_\nu) = \overline{\Lambda}\,[S_\nu(t_\nu)]$ $\quad I_\nu(\tau_{\nu\mu},\mu) \equiv I^\pm_{\nu\mu} = \Lambda_{\nu\mu}[S_\nu(\tau_{\nu\mu})]$

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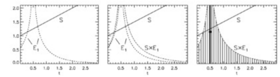
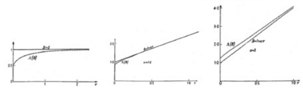
RADIATION FROM ELSEWHERE: THE Λ OPERATOR

$$J_\nu(\tau_\nu) = \Lambda_{\tau_\nu}[S_\nu(t_\nu)] \equiv \tfrac{1}{2}\int_0^\infty S_\nu(t_\nu)\,E_1(|t_\nu-\tau_\nu|)\,dt_\nu$$



Krijger 2003rtsa.book.....R



Kourganoff 1952QB801.K78......

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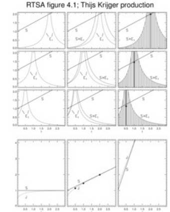
THE WORKING OF THE LAMBDA OPERATOR

RTSA figure 4.1; Thijs Krijger production



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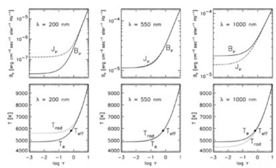
THE Λ OPERATOR FOR AN LTE-RE ATMOSPHERE

$$J_\nu(\tau_\nu) = \Lambda_{\tau_\nu}[B_\nu(t_\nu)] \equiv \tfrac{1}{2}\int_0^\infty B_\nu(t_\nu)\,E_1(|t_\nu-\tau_\nu|)\,dt_\nu$$



M.J. Krijger 1998

Conversion to formal radiation temperatures $B_\nu(T_{\rm rad}) \equiv J_\nu$ removes the wavelength dependence of the Planck function sensitivity to temperature

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SOLAR SPECTRUM FORMATION: THEORY

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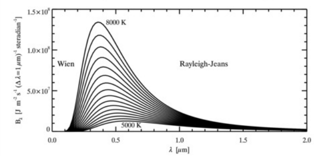
PLANCK

Planck function in intensity units

$$B_\nu(T) = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT}-1} \qquad B_\lambda(T) = \frac{2hc^2}{\lambda^5}\frac{1}{e^{hc/\lambda kT}-1} \qquad B_\sigma(T) = 2hc^2\sigma^3\frac{1}{e^{hc\sigma/kT}-1}$$

Approximations

Wien: $B_\nu(T) \approx \frac{2h\nu^3}{c^2}\,e^{-h\nu/kT}$ $\qquad$ Rayleigh-Jeans: $B_\nu(T) \approx \frac{2\nu^2 kT}{c^2}$



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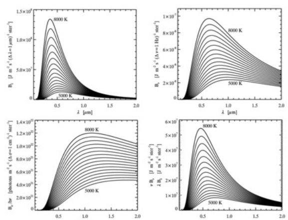
PLANCK FUNCTION VARIATIONS



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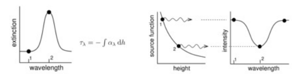
SOLAR ABSORPTION LINES AND LIMB DARKENING

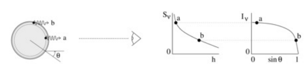
Emergent intensity without irradiation

$$I_\nu(0,\mu) = (1/\mu)\int_0^\infty S_\nu(\tau_\nu)\,e^{-\tau_\nu/\mu}\,d\tau_\nu$$

Eddington-Barbier approximation

$$\boxed{I_\nu(0,\mu) \approx S_\nu(\tau_\nu=\mu)}$$





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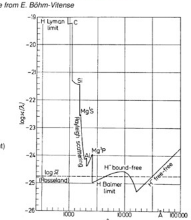
CONTINUOUS OPACITY IN THE SOLAR PHOTOSPHERE

Figure from E. Böhm-Vitense

- *bound-free*
  - optical, near-infrared: H⁻
  - UV: Si I, Mg I, Al I, Fe I (electron donors for H⁻)
  - EUV: H I Lyman; He I, He II
- *free-free*
  - infrared, sub-mm: H⁻
  - radio: H I
- *electron scattering*
  - Thomson scattering (large height)
  - Rayleigh scattering (near-UV)
- *Rosseland average*

$$\frac{1}{\overline{\kappa}} = \int_0^\infty \frac{1}{\kappa_\nu}\,\frac{dB_\nu/dT}{dB/dT}\,d\nu$$



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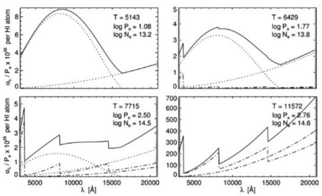
CONTINUOUS EXTINCTION H I bf, H I ff, H⁻ bf, H⁻ ff, TOTAL

after Figures 8.5 of Gray (2005), without He and with Fig. 8.5a corrected



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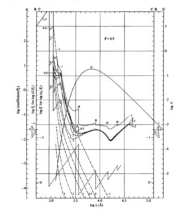
ERIKA [BOEHM-] VITENSE THESIS DIAGRAMS

Vitense 1951ZA.....28...81V   Novotny 1973itsa.book.....N
explanation: caption Fig. 8.6 RTSA and Exercise 10 RTSA



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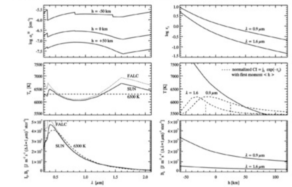
SOLAR OPTICAL AND NEAR-INFRARED CONTINUUM FORMATION

Assumed: LTE, opacity only from H I bf+ff and H⁻ bf+ff, FALC model atmosphere
Solar disk-center continuum: from Allen, *"Astrophysical Quantities"*, 1976



Does the Eddington-Barbier approximation hold?

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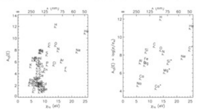
ABUNDANCES AND IONIZATION ENERGIES

nr.	element	solar abundance	χ_1	χ_2	χ_3	χ_4
1	H	1	13.598	–	–	–
2	He	7.9×10^{-2}	24.587	54.416	–	–
6	C	3.2×10^{-4}	11.260	24.383	47.887	64.492
7	N	1.0×10^{-4}	14.534	29.601	47.448	77.472
8	O	6.3×10^{-4}	13.618	35.117	54.934	77.413
11	Na	2.0×10^{-6}	5.139	47.286	71.64	98.91
12	Mg	2.5×10^{-5}	7.646	15.035	80.143	109.31
13	Al	2.5×10^{-6}	5.986	18.826	28.448	119.99
14	Si	3.2×10^{-5}	8.151	16.345	33.492	45.141
20	Ca	2.0×10^{-6}	6.113	11.871	50.91	67.15
26	Fe	3.2×10^{-5}	7.870	16.16	30.651	54.8
38	Sr	7.1×10^{-10}	5.695	11.030	43.6	57



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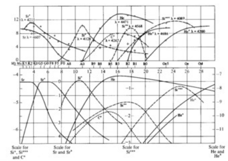
CECILIA PAYNE'S POPULATION CURVES



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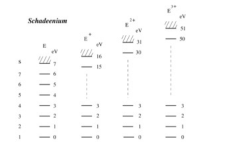
SAHA–BOLTZMANN POPULATIONS



Boltzmann distribution per ionization stage: $\frac{n_{r,s}}{N_r} = \frac{g_{r,s}}{U_r}\,\mathrm{e}^{-\chi_{r,s}/kT}$

partition function: $U_r \equiv \sum_s g_{r,s}\,\mathrm{e}^{-\chi_{r,s}/kT}$

Saha distribution over ionization stages: $\frac{N_{r+1}}{N_r} = \frac{1}{N_e}\frac{2\,U_{r+1}}{U_r}\left(\frac{2\pi m_e kT}{h^2}\right)^{3/2}\,\mathrm{e}^{-\chi_r/kT}$

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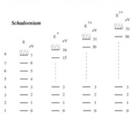
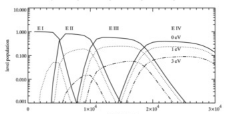
SAHA–BOLTZMANN FOR SCHADEENIUM





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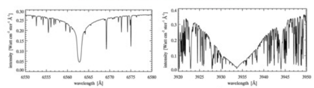
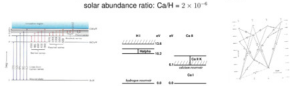
H-α AND Ca II K IN THE SOLAR SPECTRUM



solar abundance ratio: Ca/H = 2×10^{-6}



Assuming LTE at $T = 5000$ K, $P_e = 10^2$ dyne cm^{-2}:

Boltzmann H I: $\frac{n_2}{n_1} = 4.2 \times 10^{-10}$ Saha Ca II: $\frac{N_{\mathrm{Ca\,II}}}{N_{\mathrm{Ca}}} \approx 1$ $\frac{\mathrm{Ca\,II}\,(n=1)}{\mathrm{H\,I}\,(n=2)} = 8 \times 10^3$

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LTE EXTINCTION FOR Ca II H AND Hα IN FALC

Leenaarts et al. 2006A&A...449.1209L

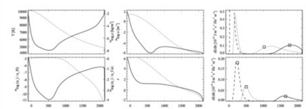

Fig. 6. Illustration explaining the difference between Hα and Ca II H wing formation. *Top left*: FALC temperature (lefthand scale, solid) and density (righthand scale, dotted). *Bottom left*: number density of the lower level of the line relative to the total number density of the atomic species, respectively for Hα (lefthand scale, solid) and for Ca II H (righthand scale, dotted). *Top center*: line center extinction coefficient for Hα (solid) and Ca II H (dotted). *Bottom center*: line center optical depth for Hα (solid) and Ca II H (dotted). *Top right*: intensity contribution function for $\Delta\lambda = 0$ (solid), −0.038 (dotted), and −0.084 nm (dashed) from line center in Hα. Squares indicate the $\tau = 1$ height. *Bottom right*: the same for Ca II H, at $\Delta\lambda = 0$ (solid), −0.024 (dotted), and −0.116 nm (dashed) from line center.

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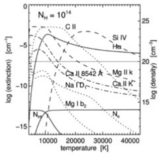
SOLAR SAHA-BOLTZMANN EXTINCTION OF STRONG LINES



- parcel of solar-composition gas with given total hydrogen density
- bottom curves: neutral hydrogen density, electron density
- other curves: line extinction for Saha-Bolzmann lower-level populations
- horizontal line: $\tau = 1$ thickness for a 100-km slab
- lower N_H: curves shift but patterns remain similar
- note: Hα hot-gas champion, Hα – 8542 crossover, Mg I b₂ – Na I D₁ crossover

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LINE BROADENING

Doppler broadening = line-of-sight Maxwell component [+ "microturbulence"]: Gaussian

$$\Delta\nu_D \equiv \frac{\nu_0}{c}\sqrt{\frac{2kT}{m}}\,[+\xi^2_{\rm micro}] \qquad \varphi(\nu-\nu_0) = \frac{1}{\sqrt{\pi}\,\Delta\nu_D}\,e^{-(\Delta\nu/\Delta\nu_D)^2}$$

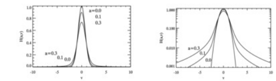
natural broadening = radiative damping from uncertainty relation: Lorentzian

$$\gamma^{\rm rad} = \gamma^{\rm rad}_l + \gamma^{\rm rad}_u = \sum_{i<l} A_{li} + \sum_{i<u} A_{ui} \qquad \psi(\nu-\nu_0) = \frac{\gamma^{\rm rad}/4\pi^2}{(\nu-\nu_0)^2 + (\gamma^{\rm rad}/4\pi)^2}$$

collisional damping: impact (Lorentzian) or quasistatic approximation (Holtsmark)
metal lines: Van der Waals, Lorentzian with $\gamma^{\rm total} = \gamma^{\rm rad} + \gamma^{\rm coll}$
hydrogen lines: linear Stark + resonance, Holtsmark

line extinction coefficient shape = convolution Gaussian ⊗ Lorentzian = Voigt function

$$\phi(\nu) = \frac{1}{\sqrt{\pi}\,\Delta\nu_D}\,H(a,v) \qquad v \equiv \frac{\nu-\nu_0}{\Delta\nu_D} \qquad a \equiv \frac{\gamma^{\rm total}}{4\pi\Delta\nu_D} \qquad H(a,v) \equiv \frac{a}{\pi}\int_{-\infty}^{+\infty}\frac{e^{-y^2}}{(v-y)^2+a^2}\,dy$$



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SOLAR SPECTRUM FORMATION: THEORY

Robert J. Rutten

https://webspace.science.uu.nl/~rutte101

start: dawn of astrophysics exercises literature 101-intro

basics: basic quantities flux intensity conservation exam constant S_ν plane-atmosphere RT EB CF+RF formation cartoons E-B exam $\Lambda(S)$

LTE 1D static: Planck EB-line-limb continuous opacity electron donors Saha-Boltzmann line broadening LTE line equations

NLTE descriptions: solar radiation processes bb equilibria Einstein coefficients line source function formal temperatures departure coefficients lasering population + transport equations

scattering: 2-level atoms sharp atom CZ demo scattering equations results

refinements: partial redistribution multi-level detours radiative cooling balancing Λ iteration

course summary: all bb pairs NLTE line cartoon equation summary key equations scattering cont & line NLTE summary cartoon homework

course finish: H I exam moral conclusion

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LTE LINES

Voigt function

$$v \equiv \frac{\nu-\nu_0}{\Delta\nu_D} \qquad a \equiv \frac{\gamma}{4\pi\Delta\nu_D} \qquad H(a,v) \equiv \frac{a}{\pi}\int_{-\infty}^{+\infty}\frac{e^{-y^2}}{(v-y)^2+a^2}\,dy \qquad \text{area in } \nu\text{: } \sqrt{\pi}\Delta\nu_D$$

line extinction coefficient in LTE

$$\sigma^l_\nu = \frac{h\nu}{4\pi}B_{lu}\,\phi(\nu) = \frac{\pi e^2}{m_e c}f_l\,\phi(\nu) = \frac{\sqrt{\pi}\,e^2 f_l}{m_e c\,\Delta\nu_D}H(a,v) \qquad A_{ul} = 6.67\times10^{13}\frac{g_l}{g_u}\frac{f_{lu}}{\lambda^2}\ \mathrm{s^{-1}}\ (\lambda \text{ in nm})$$

$$\alpha^l_\nu = \sigma^l_\nu\,n^{\rm LTE}_l\,(1-e^{-h\nu/kT}) = \sigma^l_\nu\,\frac{n_i}{\sum n_i}\,\frac{n_{ij}}{n_i}\,\frac{n_{ijk}}{n_{ij}}\,(1-e^{-h\nu/kT}) \qquad i,j,k \text{ species, stage, lower level}$$

abundance, Saha, Boltzmann

use continuum optical depth scale as reference

$$\eta_\nu \equiv \alpha^l_\nu/\alpha^c_\nu \qquad d\tau_\nu = d\tau^c_\nu + d\tau^l_\nu = (1+\eta_\nu)\,d\tau^c_\nu$$

emergent intensity at disk center in LTE

$$I_\nu(0,1) = \int_0^\infty B_\nu\,\exp(-\tau_\nu)\,d\tau_\nu = \int_0^\infty (1+\eta_\nu)\,B_\nu\,\exp\left(-\int_0^{\tau^c_\nu}(1+\eta_\nu)\,d\tau^c_\nu\right)d\tau^c_\nu$$

Eddington-Barbier

$$I_\nu(0,1) \approx B_\nu\,(T\,[\tau_\nu\!=\!1]) = B_\nu\,(T\,[\tau^c_\nu\!=\!1/(1+\eta_\nu)])$$

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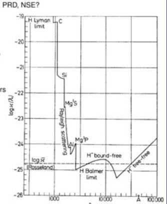
SOLAR ATMOSPHERE RADIATIVE PROCESSES

- *bound-bound* – κ_ν, S_ν: CE, LTE, NLTE, PRD, NSE?
  - neutral atom transitions
  - ion transitions
  - molecule transitions
- *bound-free* – same except always CRD
  - H⁻ optical, near-infrared
  - H I Balmer, Lyman; He I, He II
  - Fe I, Si I, Mg I, Al I = electron donors
- *free-free* – $S_\nu = B_\nu$
  - H⁻ infrared, sub-mm
  - H I mm, radio
- *electron scattering* – $S_\nu = J_\nu$
  - Thomson scattering
  - Rayleigh scattering
- *collective* – p.m.
  - cyclotron, synchrotron radiation
  - plasma radiation



E. Böhm-Vitense

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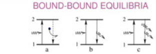
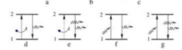
BOUND-BOUND EQUILIBRIA

line extinction:



line emissivity:



- *LTE = large collision frequency* – interior, low photosphere
  - up: mostly collisional = thermal creation (d + e)
  - down: mostly collisional = large destruction probability (a)
  - photon travel: "honorary gas particles" or negligible leak
- *NLTE, NSE = statistical equilibrium or time-dependent* – chromosphere, "TR"
  - photon travel: non-local impinging (pumping), loss (suction)
  - two-level scattering: complete/partial/angle redistribution
  - multi-level: photon conversion, sensitivity transcription
- *CE = coronal equilibrium = thin tenuous* – coronal EUV
  - up: only collisional = thermal creation (only d)
  - down: only spontaneous (only d)
  - photon travel: escape / drown / scatter bf H I, He I, He II

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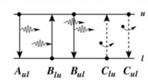
BOUND-BOUND PROCESSES AND EINSTEIN COEFFICIENTS



Spontaneous deexcitation

A_{ul} ≡ transition probability for spontaneous deexcitation from state u to state l per sec per particle in state u

Radiative excitation

$B_{lu}\overline{J}^{\varphi}_{\nu_0}$ ≡ number of radiative excitations from state l to state u per sec per particle in state l

Induced deexcitation

$B_{ul}\overline{J}^{\chi}_{\nu_0}$ ≡ number of induced radiative deexcitations from state u to state l per sec per particle in state u

Collisional excitation and deexcitation

C_{lu} ≡ number of collisional excitations from state l to state u per sec per particle in state l

C_{ul} ≡ number of collisional deexcitations from state u to state l per sec per particle in state u

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LINE SOURCE FUNCTION

RTSA 2.3.1, 2.3.2, 2.6.1

Monochromatic bb rates expressed in Einstein coefficients (per steradian, as intensity)

$n_u A_{ul}\chi(\nu)/4\pi$ — spontaneous emission; $n_u B_{ul} I_\nu \psi(\nu)/4\pi$ — stimulated emission; $n_l B_{lu} I_\nu \phi(\nu)/4\pi$ — radiative excitation; $n_u C_{ul}$, $n_l C_{lu}$ — collisional (de-)excitation

Einstein relations

$$g_u B_{ul} = g_l B_{lu} \qquad (g_u/g_l) A_{ul} = (2h\nu^3/c^2)\, B_{lu} \qquad C_{ul}/C_{lu} = (g_l/g_u)\exp(E_{ul}/kT)$$

required for TE detailed balancing with $I_\nu = B_\nu$, but hold universally

General line source function

$$j_\nu = \frac{h\nu}{4\pi} n_u A_{ul}\,\chi(\nu) \qquad \alpha_\nu = \frac{h\nu}{4\pi}\left[n_l B_{lu}\phi(\nu) - n_u B_{ul}\psi(\nu)\right] \qquad S_l = \frac{n_u A_{ul}\,\chi(\nu)}{n_l B_{lu}\phi(\nu) - n_u B_{ul}\psi(\nu)}$$

Simplified line source function

CRD: $\chi(\nu) = \psi(\nu) = \phi(\nu)$ $\qquad S_l = \dfrac{n_u A_{ul}}{n_l B_{lu} - n_u B_{ul}} = \dfrac{2h\nu^3}{c^2}\dfrac{1}{\frac{g_u n_l}{g_l n_u} - 1}$ $\qquad$ Boltzmann: $S_l = B_\nu(T)$

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FORMAL TEMPERATURES

RTSA 2.6.2

Excitation temperature

$$\frac{n_u}{n_l} \equiv \frac{g_u}{g_l}\,e^{-h\nu/kT_{\rm exc}} \qquad S^l_{\nu_0} = \frac{2h\nu_0^3}{c^2}\frac{1}{\frac{g_u n_l}{g_l n_u} - 1} = \frac{2h\nu_0^3}{c^2}\frac{1}{e^{h\nu_0/kT_{\rm exc}} - 1} = B_{\nu_0}(T_{\rm exc})$$

Ionization temperature

$$S^{\rm bf}_\nu \equiv \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT_{\rm ion}} - 1} = B_\nu(T_{\rm ion})$$

Radiation temperature

$$B_\nu(T_{\rm rad}) \equiv J_\nu$$

Brightness temperature

$$B_\nu(T_b) \equiv I_\nu$$

Effective temperature

$$\pi B(T_{\rm eff}) \equiv \sigma T_{\rm eff}^4 = \mathcal{F}_{\rm surface}$$

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POPULATION DEPARTURE COEFFICIENTS

RTSA 2.6.2

Population departure coefficients

$$b_l = n_l/n_l^{\rm LTE} \qquad b_u = n_u/n_u^{\rm LTE}$$

Line source function

$$S^l_\nu = \frac{2h\nu^3}{c^2}\frac{\psi/\varphi}{\frac{b_l}{b_u}e^{h\nu/kT} - \frac{\chi}{\varphi}} \qquad \text{CRD: } S^l_{\nu_0} = \frac{2h\nu_0^3}{c^2}\frac{1}{\frac{b_l}{b_u}e^{h\nu_0/kT} - 1}, \qquad \text{Wien: } S^l_{\nu_0} \approx \frac{b_u}{b_l}B_{\nu_0}$$

Monochromatic line extinction coefficient

$$\alpha^l_\nu = \frac{h\nu}{4\pi} b_l n_l^{\rm LTE} B_{lu}\varphi(\nu-\nu_0)\left[1 - \frac{b_u n_u^{\rm LTE} B_{ul}\,\chi}{b_l n_l^{\rm LTE} B_{lu}\,\varphi}\right] = \frac{h\nu}{4\pi} b_l n_l^{\rm LTE} B_{lu}\varphi(\nu-\nu_0)\left[1 - \frac{b_u}{b_l}\frac{\chi}{\varphi}e^{-h\nu/kT}\right]$$

$$= b_l n_l^{\rm LTE}\sigma^l_\nu\left[1 - \frac{b_u}{b_l}\frac{\chi}{\varphi}e^{-h\nu/kT}\right] = \frac{\pi e^2}{m_e c} b_l n_l^{\rm LTE} f_{lu}\varphi(\nu-\nu_0)\left[1 - \frac{b_u}{b_l}\frac{\chi}{\varphi}e^{-h\nu/kT}\right]$$

Wien: $\alpha^l_\nu \approx b_l\,[\alpha^l_\nu]_{\rm LTE}$

Total line extinction coefficient

$$\alpha^l_{\nu_0} = \frac{h\nu_0}{4\pi} b_l n_l^{\rm LTE} B_{lu}\left[1 - \frac{b_u}{b_l}e^{-h\nu_0/kT}\right] = \frac{\pi e^2}{m_e c} b_l n_l^{\rm LTE} f_{lu}\left[1 - \frac{b_u}{b_l}e^{-h\nu_0/kT}\right] \approx b_l\,[\alpha^l_{\nu_0}]_{\rm LTE}$$

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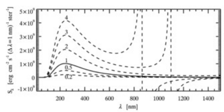
LASERING

RTSA 2.6.2

Laser regime for sufficient excess $b_u > b_l$

$$1 - (b_u/b_l)\exp(-h\nu_0/kT) < 0 \quad\Longrightarrow\quad \alpha^l_{\nu_0} < 0 \qquad S^l_{\nu_0} < 0$$

$$\frac{S^l_{\nu_0}}{B_{\nu_0}} = \frac{1 - e^{-h\nu_0/kT}}{(b_l/b_u)\left[1 - (b_u/b_l)\,e^{-h\nu_0/kT}\right]} = b_u\frac{[\alpha^l_{\nu_0}]_{\rm LTE}}{\alpha^l_{\nu_0}}$$



Wavelength variation of the NLTE source function for $T = 10\,000$ K and the specified ratios b_u/b_l. The NLTE source function scales with the Planck function (solid curve for $b_u/b_l = 1$) in the Wien part at left, but reaches the laser regime for large b_u/b_l in the Rayleigh-Jeans part at right.

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STATISTICAL EQUILIBRIUM VERSUS NON-EQUILIBRIUM EVALUATION

RTSA 2.6.1

Statistical equilibrium equations for level j

$$n_j\sum_{j\neq i}^{N} R_{ji} = \sum_{j\neq i}^{N} n_j R_{ij} \qquad R_{ji} = A_{ji} + B_{ji}\overline{J_{ji}} + C_{ji} \qquad \overline{J_{ji}} \equiv \frac{1}{4\pi}\int_0^{4\pi}\int_0^\infty I_\nu\,\phi(\nu)\,{\rm d}\nu\,{\rm d}\Omega$$

time-independent population — bb rates per particle in j — total (= mean) mean intensity for CRD

Transport equation in differential form

$$\mu\frac{{\rm d}I_\nu}{{\rm d}\tau_\nu} = I_\nu - S_\nu$$

Transport equation in integral form

$$I^-_\nu(\tau_\nu,\mu) = -\int_0^{\tau_\nu} S_\nu(t_\nu)\,e^{-(t_\nu-\tau_\nu)/\mu}\,{\rm d}t_\nu/\mu$$

$$I^+_\nu(\tau_\nu,\mu) = +\int_{\tau_\nu}^{\infty} S_\nu(t_\nu)\,e^{-(t_\nu-\tau_\nu)/\mu}\,{\rm d}t_\nu/\mu$$

1D ("plane-parallel") SE: require in $=$ out, solve these coupled equations for all wavelengths and levels of all pertinent transitions for a sufficient number of μ angles at all heights

3D non-E: evaluate the net rates for all wavelengths and levels of all pertinent transitions for a sufficient number of ϕ and ψ angles at all (x,y,z) locations as a function of time. If energetically important, couple back into the energy equation in the simulation

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SOLAR SPECTRUM FORMATION: THEORY

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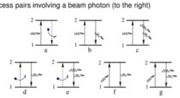
TWO-LEVEL TRANSPORT

RTSA 3.4

All two-level process pairs involving a beam photon (to the right)



Sharp-line two-level atom = monochromatic "redistribution"

$$\frac{{\rm d}I_{\nu_0}}{{\rm d}s} = \frac{h\nu_0}{4\pi}n_1\Bigg[\underbrace{-B_{12}I_{\nu_0}\frac{C_{21}}{P_{21}}}_{(a)} \underbrace{-B_{12}I_{\nu_0}\frac{A_{21}}{P_{21}}}_{(b)} \underbrace{-B_{12}I_{\nu_0}\frac{B_{21}J_{\nu_0}}{P_{21}}}_{(c)}$$

$$+\underbrace{C_{12}\frac{A_{21}}{P_{21}}}_{(d)} + \underbrace{C_{12}\frac{B_{21}I_{\nu_0}}{P_{21}}}_{(e)} + \underbrace{B_{12}J_{\nu_0}\frac{A_{21}}{P_{21}}}_{(f)} + \underbrace{B_{12}J_{\nu_0}\frac{B_{21}I_{\nu_0}}{P_{21}}}_{(g)}\Bigg]$$

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TWO-LEVEL SOURCE FUNCTION

sharp-line atom derivation: RTSA 3.4

Collisional destruction probability per extinction

$$\varepsilon_{\nu_0} \equiv \frac{\alpha^a_{\nu_0}}{\alpha^s_{\nu_0} + \alpha^a_{\nu_0}} = \frac{C_{21}}{A_{21}/[1 - \exp(-h\nu_0/kT)] + C_{21}} = \frac{C_{21}}{A_{21} + B_{21}B_{\nu_0} + C_{21}}$$

Alternate form

$$\varepsilon'_{\nu_0} \equiv \alpha^a_{\nu_0}/\alpha^s_{\nu_0} = \frac{\varepsilon_{\nu_0}}{1 - \varepsilon_{\nu_0}} = \frac{C_{21}}{A_{21}}\left[1 - e^{-h\nu_0/kT}\right]$$

Line source function

$$S^l_{\nu_0} \equiv \frac{j^l_{\nu_0}}{\alpha^l_{\nu_0}} = (1 - \varepsilon_{\nu_0})\,J_{\nu_0} + \varepsilon_{\nu_0}B_{\nu_0} = \frac{J_{\nu_0} + \varepsilon'_{\nu_0}B_{\nu_0}}{1 + \varepsilon'_{\nu_0}}$$

Complete frequency redistribution

$$S^l_{\nu_0} = (1 - \varepsilon_{\nu_0})\,\overline{J}^\varphi_{\nu_0} + \varepsilon_{\nu_0}B_{\nu_0} = \frac{\overline{J}^\varphi_{\nu_0} + \varepsilon'_{\nu_0}B_{\nu_0}}{1 + \varepsilon'_{\nu_0}}$$

Frequency-independent, but beware

$$S^{\rm tot}_\nu = \frac{\alpha^l_\nu S^l_{\nu_0} + \alpha^c_\nu S^c_\nu}{\alpha^l_\nu + \alpha^c_\nu}$$

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SUMMARY SCATTERING EQUATIONS

RTSA 4.1–4.3

Destruction probability

coherent: $\varepsilon_\nu \equiv \dfrac{\alpha^a_\nu}{\alpha^a_\nu + \alpha^s_\nu}$ $\qquad$ 2-level CRD: $\varepsilon_{\nu_0} \equiv \dfrac{\alpha^a_{\nu_0}}{\alpha^a_{\nu_0} + \alpha^s_{\nu_0}} = \dfrac{C_{21}}{C_{21} + A_{21} + B_{21}B_{\nu_0}}$

Elastic scattering

coherent: $S_\nu = (1 - \varepsilon_\nu)J_\nu + \varepsilon_\nu B_\nu$ $\qquad$ 2-level CRD: $S_{\nu_0} = (1 - \varepsilon_{\nu_0})\overline{J}_{\nu_0} + \varepsilon_{\nu_0}B_{\nu_0}$

Schwarzschild equation and Lambda operator

$$J_\nu(\tau_\nu) \equiv \frac{1}{2}\int_{-1}^{+1} I_\nu(\tau_\nu,\mu)\,{\rm d}\mu = \frac{1}{2}\int_0^\infty S_\nu(t_\nu)\,E_1(|t_\nu - \tau_\nu|)\,{\rm d}t_\nu \equiv \mathbf{\Lambda}_{\tau_\nu}[S_\nu(t_\nu)]$$

surface: $J_\nu(0) \approx \frac{1}{2}S_\nu(\tau_\nu = 1/2)$ $\qquad$ depth: $J_\nu(\tau_\nu) \approx S_\nu(\tau_\nu)$ $\qquad$ diffusion: $J_\nu(\tau_\nu) \approx B_\nu(\tau_\nu)$

Scattering in an isothermal atmosphere with constant ε_ν

coherent: $S_\nu(0) = \sqrt{\varepsilon_\nu}\,B_\nu$ $\qquad$ 2-level CRD: $S_{\nu_0}(0) = \sqrt{\varepsilon_{\nu_0}}\,B_{\nu_0}$

Thermalizaton depth

coherent: $\Lambda_\nu = 1/\varepsilon_\nu^{1/2}$ $\qquad$ Gauss profile: $\Lambda_{\nu_0} \approx 1/\varepsilon_{\nu_0}$ $\qquad$ Lorentz profile: $\Lambda_{\nu_0} \approx 1/\varepsilon^2_{\nu_0}$

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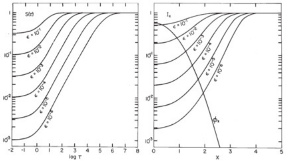
CRD RESONANT SCATTERING IN AN ISOTHERMAL ATMOSPHERE

RTSA figure 4.12; from Avrett 1965SAOSR.174..101A



- left: S/B in a plane-parallel isothermal atmosphere with constant ε for complete redistribution. The curves illustrate the $\sqrt{\varepsilon}$ law and thermalization at $\Lambda \approx 1/\varepsilon$.
- right: corresponding emergent line profiles and Gaussian extinction profile shape ϕ (only the righthand halves; $x = \Delta\lambda/\Delta\lambda_D$)

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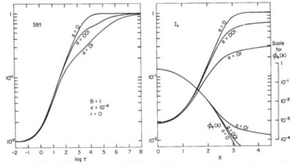
TWO-LEVEL SCATTERING FOR DIFFERENT LINE PROFILES

RTSA figure 4.12; from Avrett 1965SAOSR.174..101A



- left: S/B in a plane-parallel isothermal atmosphere with constant $\varepsilon = 10^{-4}$ for complete redistribution with three different Voigt damping parameters
- right: corresponding emergent line profiles and extinction profile shapes. The thermalization depth increases for larger damping because the extended outer wings provide deeper photon escape

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TWO-LEVEL SCATTERING WITH BACKGROUND CONTINUUM

RTSA figure 4.13; from Avrett 1965SAOSR.174..101A

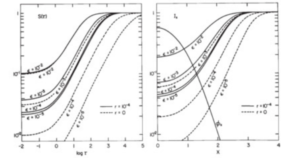

Figure 4.13: Avrett results for two-level-atom lines with complete redistribution and a background continuum. The atmosphere is isothermal. Axis labeling and parameters as for the upper panels of Figure 4.12; the extinction profile $\varphi(x)$ is again Gaussian (righthand panel). *Dashed curves:* $r \equiv \alpha^c_\nu/\alpha^l_{\nu_0}$ set to $r = 0$, describing pure resonance scattering without background continuum. *Solid curves:* $r = 10^{-4}$ or $\eta_{\nu_0} = 10^4$, describing fairly strong lines. Lack of continuum thermalization is unimportant when $r \ll \varepsilon_{\nu_0}$. Lack of collisional destruction is unimportant when $\varepsilon_{\nu_0} \ll r$. From Avrett (1965).

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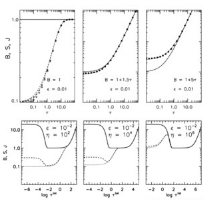
SCATTERING IN EPSILON = 0.01 ATMOSPHERES

$J_\nu(\tau_\nu) = \Lambda_{\tau_\nu}[S_\nu(t_\nu)]$ $\quad S^l_{\nu_0} = (1-\varepsilon_{\nu_0})\,J_{\nu_0} + \varepsilon_{\nu_0} B_{\nu_0}$



Krijger 2003rtsa.book.....R

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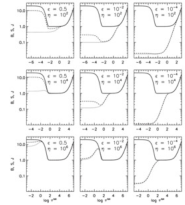
B, J, S FOR SOLAR-LIKE COHERENT LINE SCATTERING

RTSA figure 4.11; Thijs Krijger production



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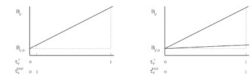
FLAT $S(\tau)$ IN STRONG LINES

Fig. 4.10 of RTSA



- RE: $B_\nu(\tau^c_\nu) \approx B_{\nu,0}\,(1 + 1.5\,\tau^c_\nu)$ at peak of emergent flux (optical)
- strong line: $\eta_\nu \equiv \alpha^l_\nu/\alpha^c_\nu >> 1$
- tau scaling in line: $\mathrm{d}\tau^{\rm tot}_\nu = \mathrm{d}\tau^c_\nu + \mathrm{d}\tau^l_\nu = (1+\eta_\nu)\,\mathrm{d}\tau^c_\nu$
- RE gradient seen by line: $B_\nu(\tau^{\rm tot}_\nu) \approx B_{\nu,0}\,(1 + 1.5/(1+\eta_\nu)\,\tau^{\rm tot}_\nu)$
- strong lines tend to obey the $\sqrt{\varepsilon}$ law: $S(\tau\!=\!1) << B[T(\tau\!=\!\Lambda)]$

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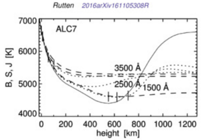
SCATTERING ULTRAVIOLET CONTINUA

Rutten 2016arXiv161105308R



- photospheric $T(h)$ gradient set in optical by RE
- ultraviolet $B[T(h)]$ much steeper from Wien
- no $B(\tau)$ flattening from strong-line extinction
- Λ operator produces $J > B$

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SOLAR SPECTRUM FORMATION: THEORY

Robert J. Rutten

https://webspace.science.uu.nl/~rutte101

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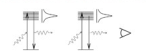
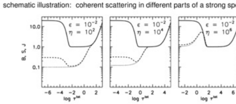
FREQUENCY COHERENCE OR REDISTRIBUTION



- Eddington: does a re-emitting atom remember at which frequency it was excited?
- yes = coherent scattering: incoming and outgoing photons same frequency
- no = complete redistribution: outgoing takes fresh sample of the probability distribution
- Doppler redistribution: coherent scattering per atom, ensemble Dopplershifts for observer
- collisional redistribution: "reshuffling while atom sits in upper state"
- schematic illustration: coherent scattering in different parts of a strong spectral line



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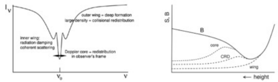
PARTIAL FREQUENCY REDISTRIBUTION PER CARTOON



- Doppler core: monofrequent ("coherent") scatttering per atom (in its moving frame); Doppler redistribution over parcel Doppler width for observer (snag: microturbulence?)
- inner damping wing: Heisenberg ⇒ coherent scattering with Doppler redistribution
- outer damping wing: at large density collisional damping ⇒ complete redistribution
- if the line is so strong that radiation damping dominates in the inner wings (high formation at low collider density) then the inner-wing photons are independent Doppler-wide ensembles with their own line source functions
- inner-wing line source function decouples deeper from the Planck function than the core source function due to smaller opacity: they represent weaker lines
- the PRD core source function decouples further out than for complete redistribution because core photons cannot escape from deeper layers via occasional wing sampling

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PARTIAL REDISTRIBUTION CLASSIC

Hummer 1962MNRAS.125...21H

NON-COHERENT SCATTERING

I. The Redistribution Functions with Doppler Broadening

David G. Hummer

Summary

In this paper we obtain a very general redistribution function for the physically realistic situations in which scattering, according to an arbitrary redistribution function and an arbitrary phase function in the atom's rest frame, is further modified by the Doppler effect. We obtain explicit formulae for the redistribution functions in four cases. They are, with the Roman numeral which will subsequently identify them:

Zero natural line width (I).
Radiation damping with coherence in the atom's rest frame (II).
Radiation and collision damping with complete redistribution in the atom's frame (III).
Resonance scattering (IV).

$$R_{\mathrm{I}}(x,\mathbf{n};x',\mathbf{n}') = \frac{g(\mathbf{n},\mathbf{n}')}{4\pi^2\sin\gamma}\exp\left[-x'^2-(x-x'\cos\gamma)^2\csc^2\gamma\right]$$

$$R_{\mathrm{II}}(x,\mathbf{n};x',\mathbf{n}') = \frac{g(\mathbf{n},\mathbf{n}')}{4\pi^2\sin\gamma}\exp\left[-\left(\frac{x-x'}{2}\right)^2\csc^2\left(\frac{\gamma}{2}\right)\right]H\left(\sigma\sec\frac{\gamma}{2},\frac{x+x'}{2}\sec\frac{\gamma}{2}\right)$$

$$R_{\mathrm{III}}(x,\mathbf{n};x',\mathbf{n}') = \frac{g(\mathbf{n},\mathbf{n}')}{4\pi^2\sin\gamma}\frac{\sigma}{\pi}\int_{-\infty}^{\infty}\frac{e^{-y^2}}{(x-y)^2+\sigma^2}H(\sigma\csc\gamma,x'\csc\gamma-y\,\mathrm{ctn}\,\gamma)\,dy$$

$$R_{\mathrm{IV}}(x,\mathbf{n};x',\mathbf{n}') = \frac{g(\mathbf{n},\mathbf{n}')}{2\pi^2\sin\gamma}\frac{\sigma_i\sec\gamma/2}{\pi}\times\int_{-\infty}^{\infty}\frac{e^{-y^2}H(\sigma_j\csc\gamma/2,\,y\,\mathrm{ctn}\,\gamma/2-x\csc\gamma/2)\,dy}{[(x-x')\sec\gamma/2-2y]^2+(\sigma_i\sec\gamma/2)^2}$$

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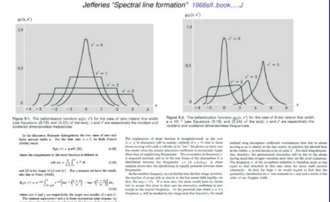
CLEAREST EXPLANATION SOFAR

Jefferies "Spectral line formation" 1968slf..book.....J



upshot: Doppler-wide core around observed line center, in far natural-damping wings Doppler-broadened coherency

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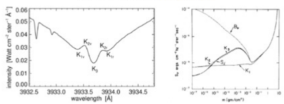
FORMATION OF Ca II K WITH PARTIAL REDISTRIBUTION

Shine, Milkey, Mihalas 1975ApJ...199..724S



- left: classic naming of Ca II K reversal pattern features (the only non-Gaussian Fraunhofer-line cores!)
- right: in CRD $S^l_{\nu_0}$ (solid curve) maps the minimum temperature into the Ca II K_1 dips
- PRD S^l_ν departs individually from B_λ for each ensemble of non-redistributed photons
- no such 1D explanation for the $K_{2V} > K_{2R}$ and $K_{1V} > K_{1R}$ asymmetries
- actually, the average profile is dominated by acoustic shocks in internetwork and magnetic concentrations in network
- the Wilson-Bappu effect (stellar core width – luminosity correlation) remains unexplained

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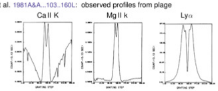
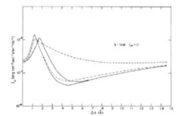
MAJOR PRD LINES

Lemaire et al. 1981A&A...103..160L: observed profiles from plage



Milkey & Mihalas 1974ApJ...192..769M: computed half Mg II k profiles for PRD



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RECENT DEVELOPMENTS IN PRD LINE SYNTHESIS

- *RH code: Uitenbroek* 2001ApJ...557..389U
  - Rybicky & Hummer: not $\Lambda(S)$ but $\Psi(j)$ iteration; preconditioning
  - overlapping lines
  - 1D, 2D, 3D, spherical versions
- *RH 1.5D: Pereira & Uitenbroek* 2015A&A...574A...3P
  - 1.5D = column-by-column
  - massively parallel
  - also molecular lines (but Kurucz lines in LTE)
- *angle-dependent redistribution: Leenaarts et al.* 2012A&A...543A.109L
  - good summary PRD theory and equations
  - non-stationary atmosphere requires angle-dependent PRD
  - hybrid approximation: transform to gas parcel frame, assume angle-averaged PRD (≈ angle dependent from deep isotropy), transform back
- *towards Bifrost PRD: Sukhorukov & Leenaarts* 2017A&A...597A..46S
  - hybrid approximation for small memory
  - linear frequency interpolation for speed
  - 252×252×496 grid, 1024 CPUs: 2 days for Mg II k ≈ doable
- *next: 3D PRD with multigrid (Bjørgen & Leenaarts* 2017A&A...599A.118B*)*

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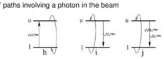
MULTI-LEVEL DETOURS

All "interlocking" paths involving a photon in the beam



Detour source function with detour transition probabilities D_{ul} D_{lu}

$$S^{\rm d}_{\nu_0} = \frac{2h\nu_0^3}{c^2}\frac{1}{\frac{g_u}{g_l}\frac{D_{ul}}{D_{lu}}-1} \equiv \frac{2h\nu_0^3}{c^2}\frac{1}{e^{h\nu_0/kT_{\rm d}}-1} = B_{\nu_0}(T_{\rm d})$$

Collision and conversion photon-loss probabilities for sharp-line atoms

$$\varepsilon_{\nu_0} \equiv \frac{\alpha^{\rm a}_{\nu_0}}{\alpha^{\rm s}_{\nu_0}+\alpha^{\rm a}_{\nu_0}+\alpha^{\rm d}_{\nu_0}} = \frac{C_{ul}\left(1-e^{-h\nu_0/kT}\right)}{A_{ul}+C_{ul}\left(1-e^{-h\nu_0/kT}\right)+D_{ul}\left(1-e^{-h\nu_0/kT_{\rm d}}\right)}$$

$$\eta_{\nu_0} \equiv \frac{\alpha^{\rm d}_{\nu_0}}{\alpha^{\rm s}_{\nu_0}+\alpha^{\rm a}_{\nu_0}+\alpha^{\rm d}_{\nu_0}} = \frac{D_{ul}\left(1-e^{-h\nu_0/kT_{\rm d}}\right)}{A_{ul}+C_{ul}\left(1-e^{-h\nu_0/kT}\right)+D_{ul}\left(1-e^{-h\nu_0/kT_{\rm d}}\right)}$$

Line source function

$$S^l_{\nu_0} = (1-\varepsilon_{\nu_0}-\eta_{\nu_0})\,J_{\nu_0} + \varepsilon_{\nu_0}B_{\nu_0}(T) + \eta_{\nu_0}B_{\nu_0}(T_{\rm d})$$

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ALTERNATE NOTATION IN THE (CLASSICAL) LITERATURE

E.g., Jefferies "Spectral Line Formation" 1968slf..book.....J

Normalized photon destruction and photon conversion

$$\varepsilon'_{\nu_0} \equiv \frac{\alpha^{\rm a}_{\nu_0}}{\alpha^{\rm s}_{\nu_0}} = \frac{\varepsilon_{\nu_0}}{1-\varepsilon_{\nu_0}-\eta_{\nu_0}} = \frac{C_{ul}\left(1-e^{-h\nu_0/kT}\right)}{A_{ul}}$$

$$\eta'_{\nu_0} \equiv \frac{\alpha^{\rm d}_{\nu_0}}{\alpha^{\rm s}_{\nu_0}} = \frac{\eta_{\nu_0}}{1-\varepsilon_{\nu_0}-\eta_{\nu_0}} = \frac{D_{ul}\left(1-e^{-h\nu_0/kT_{\rm d}}\right)}{A_{ul}}$$

Extinction coefficient

$$\alpha^l_{\nu_0} = \alpha^{\rm s}_{\nu_0}\left(1+\varepsilon'_{\nu_0}+\eta'_{\nu_0}\right)$$

Line source function

$$S^l_{\nu_0} = \frac{J_{\nu_0} + \varepsilon'_{\nu_0}B_{\nu_0}(T) + \eta'_{\nu_0}B_{\nu_0}(T_{\rm d})}{1+\varepsilon'_{\nu_0}+\eta'_{\nu_0}}$$

Complete redistribution

$$S^l_{\nu_0} = (1-\varepsilon_{\nu_0}-\eta_{\nu_0})\,\overline{J}^\varphi_{\nu_0} + \varepsilon_{\nu_0}B_{\nu_0}(T) + \eta_{\nu_0}B_{\nu_0}(T_{\rm d}) = \frac{\overline{J}^\varphi_{\nu_0} + \varepsilon'_{\nu_0}B_{\nu_0}(T) + \eta'_{\nu_0}B_{\nu_0}(T_{\rm d})}{1+\varepsilon'_{\nu_0}+\eta'_{\nu_0}}$$

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82 SOLAR SPECTRUM FORMATION: THEORY

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83 RADIATIVE COOLING

RTSA 7.3.2

Radiative equilibrium condition

$$\begin{aligned}\Phi_{\rm tot}(z) &\equiv \frac{{\rm d}\mathcal{F}_{\rm rad}(z)}{{\rm d}z} = 0\\ &= 4\pi \int_0^\infty \alpha_\nu(z)\,[S_\nu(z) - J_\nu(z)]\,{\rm d}\nu\\ &= 2\pi \int_0^\infty \int_{-1}^{+1} [j_{\nu\mu}(z) - \alpha_{\nu\mu}(z)\,I_{\nu\mu}(z)]\,{\rm d}\mu\,{\rm d}\nu\end{aligned}$$

Net radiative cooling in a two-level atom gas

$$\begin{aligned}\Phi_{ul} &= 4\pi\alpha^l_{\nu_0}(S^l_{\nu_0} - \overline{J}_{\nu_0})\\ &= 4\pi j^l_{\nu_0} - 4\pi\alpha^l_{\nu_0}\overline{J}_{\nu_0}\\ &= h\nu_0\,[n_u(A_{ul} + B_{ul}\overline{J}_{\nu_0}) - n_l B_{lu}\overline{J}_{\nu_0}]\\ &= h\nu_0\,[n_u R_{ul} - n_l R_{lu}]\end{aligned}$$

Net radiative cooling in a one-level-plus-continuum gas

$$\Phi_{ci} = 4\pi\, n_i^{\rm LTE} b_c \int_{\nu_0}^\infty \sigma_{ic}(\nu)\left[B_\nu\left(1 - {\rm e}^{-h\nu/kT}\right) - \frac{b_i}{b_c}J_\nu\left(1 - \frac{b_c}{b_i}\,{\rm e}^{-h\nu/kT}\right)\right]{\rm d}\nu$$

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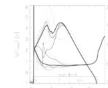
84 DETAILED BALANCING



Hydrogen ionization/recombination relaxation timescale throughout the solar-like shocked Radyn atmosphere. The timescale for settling to equilibrium at the local temperature is very long, 15–150 min, in the chromosphere but much shorter, only seconds, in shocks in which hydrogen partially ionizes.
Carlsson & Stein 2002ApJ...572..626C

net radiative and collisional downward rates (Wien approximation)

$$n_u R_{ul} - n_l R_{lu} \approx \frac{4\pi}{h\nu_0}\, n_l^{\rm LTE}\, b_u\, \sigma^l_{\nu_0}\left(B_{\nu_0} - \frac{b_l}{b_u}\overline{J}_{\nu_0}\right) \quad \text{zero for } S = \overline{J}, \text{ no heating/cooling}$$

$$n_u C_{ul} - n_l C_{lu} = n_l C_{lu}\left(\frac{b_u}{b_l} - 1\right) = b_u n_l^{\rm LTE} C_{lu}\left(1 - \frac{b_l}{b_u}\right) \quad \text{zero for } b_u = b_l, \text{ LTE } S^l$$

dipole approximation for atom collisions with electrons (Van Regemorter 1962)

$$C_{ul} \approx 2.16\left(\frac{E_{ul}}{kT}\right)^{-1.68} T^{-3/2}\frac{g_l}{g_u}\,N_e\,f$$

Einstein relation

$$C_{lu} = C_{ul}\frac{g_l}{g_u}\,{\rm e}^{-E_{ul}/kT}$$

C_{ul} is not very temperature sensitive (any collider will do); C_{lu} has Boltzmann sensitivity

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85 LAMBDA ITERATION

Lambda operator

$$J_\nu(\tau_\nu) = \Lambda_\nu[S_\nu(t)]$$

Two-level coherent scattering

$$S^l_\nu(\tau_\nu) = (1 - \varepsilon_\nu(\tau_\nu))\,\Lambda_\nu[S^l_\nu(t_\nu)] + \varepsilon_\nu(\tau_\nu)B_\nu(\tau_\nu)$$

Drop indices

$$S = (1 - \varepsilon)\,\Lambda_\nu[S] + \varepsilon B$$

$$S = (1 - (1 - \varepsilon)\,\Lambda)^{-1}\,[\varepsilon B]$$

Iteration instead of inversion

$$S^{(n+1)} = (1 - \varepsilon)\,\Lambda[S^{(n)}] + \varepsilon B$$

Convergence

$$S^{(n+1)} - S^{(n)} = (1 - \varepsilon)\,\Lambda_\nu[S^{(n)}] + \varepsilon B - S^{(n)}$$

Large τ, small ε

$$S^{(n+1)} - S^{(n)} \approx \Lambda_\nu[S^{(n)}] - S^{(n)} \approx S^{(n)} - S^{(n)} \approx 0$$

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86 ACCELERATED LAMBDA ITERATION

Operator splitting (Cannon): define Λ^* as a valid but fast approximation

$$\Lambda_\nu = \Lambda^* + (\Lambda_\nu - \Lambda^*)$$

Still exact

$$J_\nu = \Lambda^*_\nu[S] + (\Lambda_\nu - \Lambda^*_\nu)[S]$$

Iteration inserting $n+1$ also on the righthand side

$$S^{(n+1)} = (1 - \varepsilon)\,\Lambda^*[S^{(n+1)}] + (1 - \varepsilon)(\Lambda_\nu - \Lambda^*)[S^{(n)}] + \varepsilon B$$

Reshuffle

$$S^{(n+1)} - (1 - \varepsilon)\,\Lambda^*[S^{(n+1)}] = (1 - \varepsilon)\,\Lambda_\nu[S^{(n)}] + \varepsilon B - (1 - \varepsilon)\,\Lambda^*[S^{(n)}] = S^{\rm FS} - (1 - \varepsilon)\,\Lambda^*[S^{(n)}]$$

Inversion of only the approximate operator (FS = formal solution)

$$S^{(n+1)} = (1 - (1 - \varepsilon)\,\Lambda^*)^{-1}\left[S^{\rm FS} - (1 - \varepsilon)\,\Lambda^*[S^{(n)}]\right]$$

Convergence

$$S^{(n+1)} - S^{(n)} = (1 - (1 - \varepsilon)\,\Lambda^*)^{-1}\left[S^{\rm FS} - S^{(n)}\right]$$

Acceleration

$$(1 - (1 - \varepsilon)\,\Lambda^*)^{-1} \approx 1/\varepsilon$$

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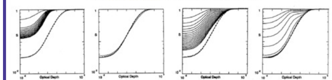
87 LAMBDA ITERATION EXAMPLES

Auer 1991sabc.conf....9A (Crivellari, Hummer, Hubený)



- isothermal semi-infinite atmosphere
- constant $\varepsilon_{\nu_0} = 10^{-3}$, complete redistribution, Gaussian profile
- display = 20 successive S^l estimates + correct S^l

- A: classical Λ iteration
- B: ALI with Scharmer operator (local Eddington-Barbier along LOS)
- C: ALI with the diagonal of the Λ matrix
- D: idem with conjugate vector acceleration

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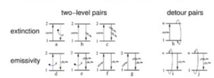
89 LINE FORMATION AS SEEN BY THE ATOM



- *pair combinations*
  - beam of interest to the right
  - a / d + e = collisional destruction / creation of beam photons
  - b + h / f + i + j scattering & detour photons out / into beam (c, g cancel)
- *equilibria*
  - LTE: a + d + e dominate; bb Boltzmann $f(T)$, bf Saha $f(T,N_e)$
  - CE: d only; bb $f(T,N_e)$, bf $f(T)$
  - NLTE, NSE: scattering and/or detours important; bb and bf $f(T, N_e, , \overline{J}_{ul}, \overline{J}_{ij}, \overline{J}_{ic})\,[t]$
- *line extinction and line source function*
  - $\alpha^l = \alpha^a + \alpha^s + \alpha^d$ absorption + scattering + detour extinction
  - $\varepsilon \equiv \alpha^a/\alpha^l$ destruction probability $\eta \equiv \alpha^d/\alpha^l$ detour probability
  - $S^l = (1 - \varepsilon - \eta)\,\overline{J} + \varepsilon B(T) + \eta S^d$ $\overline{J}$: mean mean intensity S^d: all detours

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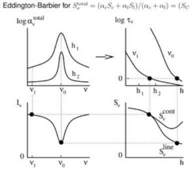
90 REALISTIC SOLAR ABSORPTION LINE

- extinction: bb peak in $\eta_\nu \equiv \alpha_l/\alpha_c$ becomes lower and narrower at larger height
- optical depth: $\tau_\nu \equiv -\int \alpha^{\rm total}_\nu\,{\rm d}h$ increases nearly log-linearly with geometrical depth
- source function: split for line (bb) and continuous (bf, ff, electron scattering) processes
- intensity: Eddington-Barbier for $S^{\rm total}_\nu = (\alpha_c S_c + \alpha_l S_l)/(\alpha_c + \alpha_l) = (S_C + \eta_\nu S_l)/(1 + \eta_\nu)$



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BASIC RADIATIVE TRANSFER EQUATIONS

last page RTSA 2003rtsa.book.....R

specific intensity	$I_\nu(\vec{r},\vec{l},t)$ erg cm^{-2} s^{-1} Hz^{-1} ster^{-1}
emissivity	j_ν erg cm^{-3} s^{-1} Hz^{-1} ster^{-1}
extinction coefficient	α_ν cm^{-1} σ_ν cm^2 part^{-1} κ_ν cm^2 g^{-1}
source function	$S_\nu = \sum j_\nu / \sum \alpha_\nu$
radial optical depth	$\tau_\nu(z_0) = \int_{z_0}^{\infty} \alpha_\nu \, dz$
plane-parallel transport	$\mu \, dI_\nu / d\tau_\nu = I_\nu - S_\nu$
thin cloud	$I_\nu = I_0 + (S_\nu - I_0)\,\tau_\nu$
thick emergent intensity	$I_\nu^+(0,\mu) = \int_0^\infty S_\nu(\tau_\nu)\, e^{-\tau_\nu/\mu}\, d\tau_\nu/\mu$
Eddington-Barbier	$I_\nu^+(0,\mu) \approx S_\nu(\tau_\nu = \mu)$
mean mean intensity	$\overline{J}_{\nu_0} = \frac{1}{2}\int_0^\infty \int_{-1}^{+1} I_\nu\, \varphi(\nu-\nu_0)\, d\mu\, d\nu$
photon destruction	$\varepsilon_\nu = \alpha_\nu^a/(\alpha_\nu^a + \alpha_\nu^s) \approx C_{ul}/(A_{ul} + C_{ul})$
complete redistribution	$S_{\nu_0}^l = (1-\varepsilon_{\nu_0})\,\overline{J}_{\nu_0} + \varepsilon_{\nu_0} B_{\nu_0}$
isothermal atmosphere	$S_{\nu_0}(0) = \sqrt{\varepsilon_{\nu_0}}\, B_{\nu_0}$

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KEY LINE FORMATION EQUATIONS

population departure coefficients

$b_l = n_l / n_l^{\rm LTE}$ $\qquad b_u = n_u / n_u^{\rm LTE}$

Zwaan: $n^{\rm LTE}$ = Saha-Boltzmann fraction of $N_{\rm el}$ $\qquad$ Harvard: n/n_c (main stages $\approx 1/b_c$)

general line extinction and line source function

$$\alpha_\lambda^l = \frac{\pi e^2}{m_e c}\frac{\lambda^2}{c}\, b_l \frac{n_l^{\rm LTE}}{N_{\rm el}} N_{\rm H} A_{\rm el} f_{lu}\, \varphi \left[1 - \frac{b_u}{b_l}\frac{\chi}{\varphi}\, e^{-hc/\lambda kT}\right] \qquad S_\lambda^l = \frac{2hc^2}{\lambda^5}\frac{\psi/\varphi}{\frac{b_l}{b_u}\, e^{hc/\lambda kT} - \frac{\chi}{\varphi}}$$

CRD approximation: $\psi = \chi = \varphi$ $\qquad$ Wien approximation: neglect stimulated parts

$\alpha^l \approx b_l\, \alpha^{\rm LTE}$ $\qquad S^l \approx (b_u/b_l)\, B(T)$

PRD: Lyα, Mg II h & k, Ca II H & K, strong UV $\qquad$ Wien: up to Hα ($\lambda T = hc/k$ at 21 900 K)

probabilities per extinction of collisional photon destruction and of detour photon conversion

$$\varepsilon \equiv \frac{\alpha^a}{\alpha^s + \alpha^a + \alpha^d} \qquad \eta \equiv \frac{\alpha^d}{\alpha^s + \alpha^a + \alpha^d}$$

line source function (for CRD, monofrequent for PRD)

$$S^l = (1 - \varepsilon - \eta)\,\overline{J} + \varepsilon B(T) + \eta S^d$$

"source" = local addition of new photons into beam per local extinction in terms of energy

$\overline{J} \equiv (1/4\pi)\int\int I\varphi\, d\Omega\, d\lambda$ reservoir $\qquad \varepsilon B$ thermal creation $\qquad \eta S^d$ detour production

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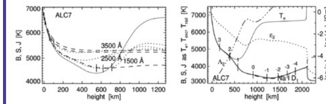
ULTRAVIOLET b-f SCATTERING VERSUS OPTICAL b-b SCATTERING



- *scattering ultraviolet continua*
  - scatter outward from deep photosphere
  - $B_\lambda(\tau_\lambda)$ steeper than defining optical RE gradient $B_{5000}(\tau_{5000}) \sim 1 + 1.5\,\tau_{5000}$
  - source function follows J, not steep drop in B
- *scattering optical line (Na I D_1)*
  - scatters outward from upper photosphere
  - optical depth scale compressed compared to $\tau_{5000} \Rightarrow B_\lambda(\tau_\lambda) \approx$ flat $\sim$ "isothermal"
  - source function doesn't care that temperature rises again

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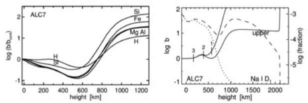
ULTRAVIOLET b-f SCATTERING VERSUS OPTICAL b-b SCATTERING



- *scattering ultraviolet continua*
  - $J - S$ translates into standard dip + rise pattern
  - photospheric minority-species lines have $b_{\rm cont} \approx 1$ and b_l extinction depletion
  - H I: Balmer continuum has same pattern in $b_2/b_{\rm cont}$ (H I top $\sim$ neutral metal)
- *scattering optical line (Na I D_1)*
  - no photospheric dip because alkalis suffer photon suction: photon-loss replenishment from population reservoir in low-lying continuum
  - steep b_l increase from ultraviolet overionization $\Rightarrow B_\lambda(\tau_\lambda) \approx$ flat $\sim$ "isothermal"
  - b_u/b_l split characteristic for scattering lines

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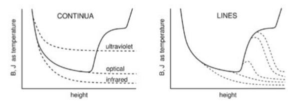
SUMMARY 1D SCATTERING SOURCE FUNCTIONS



- *continua*
  - optical: $J \approx B$ for radiative equilibrium
  - ultraviolet: $S \approx J > B \rightarrow$ overionization of minority neutrals
  - infrared: $J < B$ but J doesn't matter since H$_{\rm ff}^-$ and H$_{\rm ff}$ have $S = B$
- *lines*
  - $dB/d\tau = dB/d(\tau^c + \tau^l)$ much less steep, so closer to isothermal $S \approx \sqrt{\varepsilon}\, B$
  - for stronger lines S sees more of the model chromosphere
  - PRD lines have frequency-dependent core-to-wing $S \approx J$ curves like these

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SOLAR SPECTRUM FORMATION: THEORY

Robert J. Rutten

https://webspace.science.uu.nl/~rutte101

start: dawn of astrophysics exercises literature 101-intro

basics: basic quantities flux intensity conservation exam constant S_ν plane-atmosphere RT EB CF+RF formation cartoons E-B exam $\Lambda(S)$

LTE 1D static: Planck EB-line-limb continuous opacity electron donors Saha-Boltzmann line broadening LTE line equations

NLTE descriptions: solar radiation processes bb equilibria Einstein coefficients line source function formal temperatures departure coefficients lasering population + transport equations

scattering: 2-level atoms sharp atom CZ demo scattering equations results

refinements: partial redistribution multi-level detours radiative cooling balancing Λ iteration

course summary: all bb pairs NLTE line cartoon equation summary key equations scattering cont & line NLTE summary cartoon homework

course finish: H I exam moral conclusion

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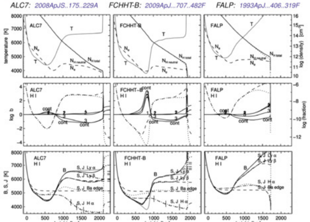
EXPLAIN EVERYTHING – INCLUDING SIMILARITIES AND DIFFERENCES

ALC7: 2008ApJS..175..229A FCHHT-B: 2009ApJ...707..482F FALP: 1993ApJ...406..319F



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WHO WANTS TO KNOW WHAT WHAT FOR?

- *optically thin cloud modeler*  courtesy Tom Berger
  - $I_\nu(\tau_\nu) = I_\nu(0)\, e^{-\tau_\nu} + \int_0^{\tau_\nu} S_\nu(t_\nu)\, e^{-(\tau_\nu - t_\nu)}\, dt_\nu \approx I_\nu(0)\, e^{-\tau_\nu(D)} + S_\nu\,(1 - e^{-\tau_\nu(D)})$
  - off limb: $I_\nu(0) = 0$ but how do I solve confusion?
  - on disk: how do I define the unseen $I_\nu(0)$?
- *optically thick Eddington-Barbier inverter* courtesy Gene Avrett
  - $I_\nu^+(\tau_\nu = 0, \mu) = \int_0^\infty S_\nu(t_\nu)\, e^{-t_\nu/\mu}\, dt_\nu/\mu \approx S_\nu(\tau_\nu = \mu)$
  - can I get away with $\tau_\nu = \tau_\nu^{\rm LTE}$ and $S_\nu = B_\nu$?
  - at what height does my line form and how does it tell me $T, N_e, \vec{v}, \vec{B}$?
- *excitable atom in the solar atmosphere*  courtesy Mats Carlsson
  - what colliders and photons are available for my excitation?
  - shall I emit or extinct a photon in the observer's direction?
  - do I muck with coherency?

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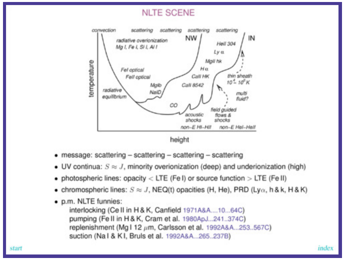

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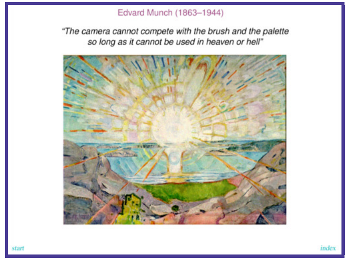

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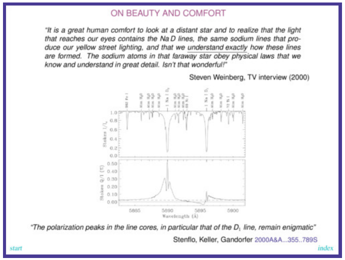

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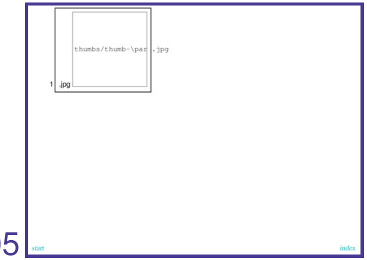

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