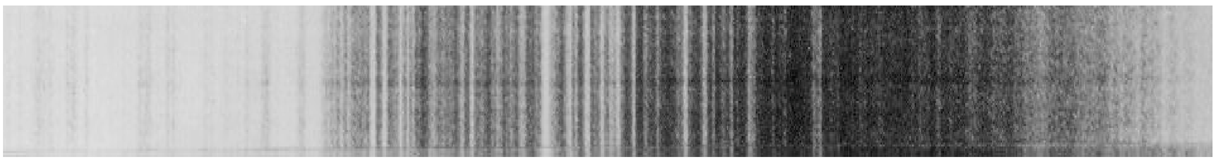
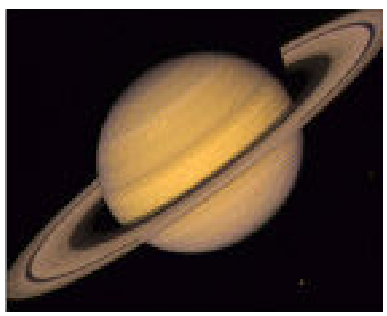
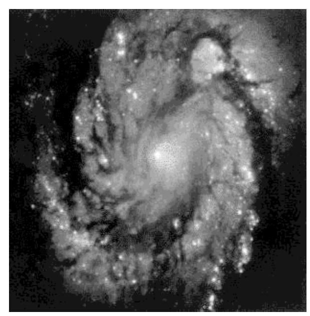
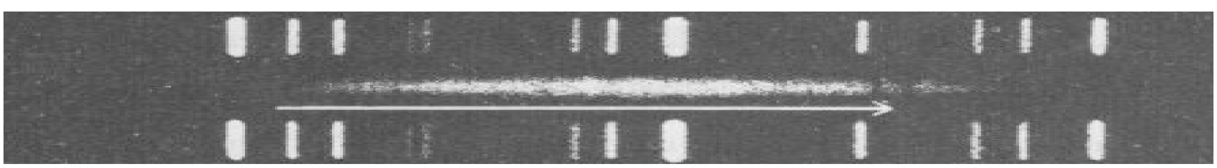
SPECTRAL LINES AS DISTANT MEASUREMENT TOOLS









R.J. Rutten
G.H. Janssen
P.R. den Hartog
J. Meijer

Sterrekundig Instituut Utrecht

First Dutch edition: October 17, 1999, Masterclass Physics & Astronomy, Utrecht University
First English edition: November 9, 2007, International Festival for Astronomy, Rhijnauwen
This web edition: July 31, 2015, `http://www.staff.science.uu.nl/~rutte101`

Contents

Marcel G.J. Minnaert (Brugge 1893 — Utrecht 1970) was a Flemish biologist who after World War I went to The Netherlands, first becoming a physicist and then director of the Utrecht Observatory "Sterrenwacht Sonnenborgh" in 1937. His most famous book is "The Nature of Light and Colour in the Open Air", later reprinted as "Light and Color in the Outdoors".

Introduction: spectral lines as measuring instrument

Astronomical research is necessarily indirect. One cannot stick a thermometer into a star to monitor its temperature while heating it: direct experiments as the ones performed by physicists in laboratories are impossible in astronomy.

Our understanding of any object in the cosmos is based on its radiation. Fortunately, this is a very rich information carrier. Electromagnetic waves propagate as fast as can (no signal exceeds the velocity of light $c \approx 300,000$ km/s in vacuum) and encompass a wide spectrum in energy (or frequency or wavelength: $E = h\nu = hc/\lambda$ with E the photon or wave packet energy, h the Planck constant 6.626068×10^{-34} m^2 kg s^{-1}, ν the frequency and λ the corresponding wavelength). Our eyes detect only a tiny part, between the rainbow colors violet ($\lambda = 400$ nm $= 4 \times 10^{-7}$ m) and red ($\lambda = 800$ nm), but the full electromagnetic spectrum stretches from energetic gamma radiation ($\lambda < 10^{-10}$ m) to long-wavelength radio waves ($\lambda > 1$ cm).

Such spectra from distant objects contain spectral lines caused by atoms and molecules within the object. They encode the spectrum with information about the local conditions (chemical composition, temperature, density, magnetic fields, motions) at microscopic scale. In addition, the lines get Doppler-shifted if the object moves towards us or away from us. This informative encoding travels faithfully along with the radiation to the observer on Earth and makes spectral lines the principal measuring tools of astrophysics.

These assignments let you employ spectral lines from a variety of objects in a variety of applications:

- you re-enact historical stellar classification;
- you study the rotation of Saturn and its rings;
- you map our environment in the Milky Way;
- you measure the expansion of the Universe.

The second and third come from *"Practical work in elementary astronomy"* by M.G.J. Minnaert. He was one of the founders of astronomical spectral-line analysis, especially for the solar spectrum by using the solar telescope and spectrograph still present at Museum Sterrenwacht Sonnenborgh in Utrecht. He also spent much effort on developing astronomy courses. His book on practical work contains forty assignments as the ones here. Most require extensive manual measurements and calculations, old-style research techniques that are outdated since we now use computers for everything. We have minimized such work in these assignments.

1 Spectral classification

In this assignment you are an astronomer at the end of the nineteenth century. Photographic spectrograms of stars were collected in large quantities, but nobody knew the cause of the spectral lines in them other than that they indicated the presence of the corresponding elements.

Your role is that of Annie Cannon, who at the Harvard College Observatory classified a quarter-million spectrograms in the scheme she devised while doing so: the O – B – A – F – G – K – M scale (*"Oh, Be A Fine Girl, Kiss Me!"*) still in use in astronomy.

Figure 1: Annie Jump Cannon (1863 – 1941) while classifying a stellar spectrogram.

Figure 3 contains a collection of stellar spectrum photographs, comparable to the spectrograms used by Annie Cannon and her collaborators at Harvard. They are negatives. Stellar spectral lines are usually absorption lines, appearing dark on a bright background continuum.

- Study the spectrograms. You are the first astronomer doing this. You appreciate that the stellar distances and the exposures differ, making some spectrograms darker than others. You also appreciate that faster-rotating stars can produce wider lines through the Doppler effect (how?). However, you have no clue as to what causes stellar spectral lines and governs their appearance. But, like a biologist, you think it useful to classify them into some order.
- Cut the page into strips, one per spectrogram. Sort these into an order that you deem significant.
- Compare your order to the one of your neighbor. Try to reach consensus.
- Compare your order to the official one available at `http://www.staff.science.uu.nl/~rutte101`.
- Compare your ordering criteria to Annie Cannon's description of her classification scheme in the first pages of her "Classification of 1,477 stars by means of their photographic spectra", available at ADS (bibcode 1912AnHar..56...65C).

Annie Cannon called the stars with few lines (her type O) "early", the ones with many lines (type M) "late", as if they get more wrinkles with age. These terms are still in use. However, the variety in the appearance of the lines in the spectrograms that you ordered has nothing to do with the age of a star, but is primarily set by the stellar surface temperature: cooler stars show more lines. The early stars are the hottest ones. The gas pressure and differences in chemical composition play only secondary roles. The Hertzsprung-Russell diagram (Figure 2) turned out to be a graph of stellar luminosity against surface temperature.

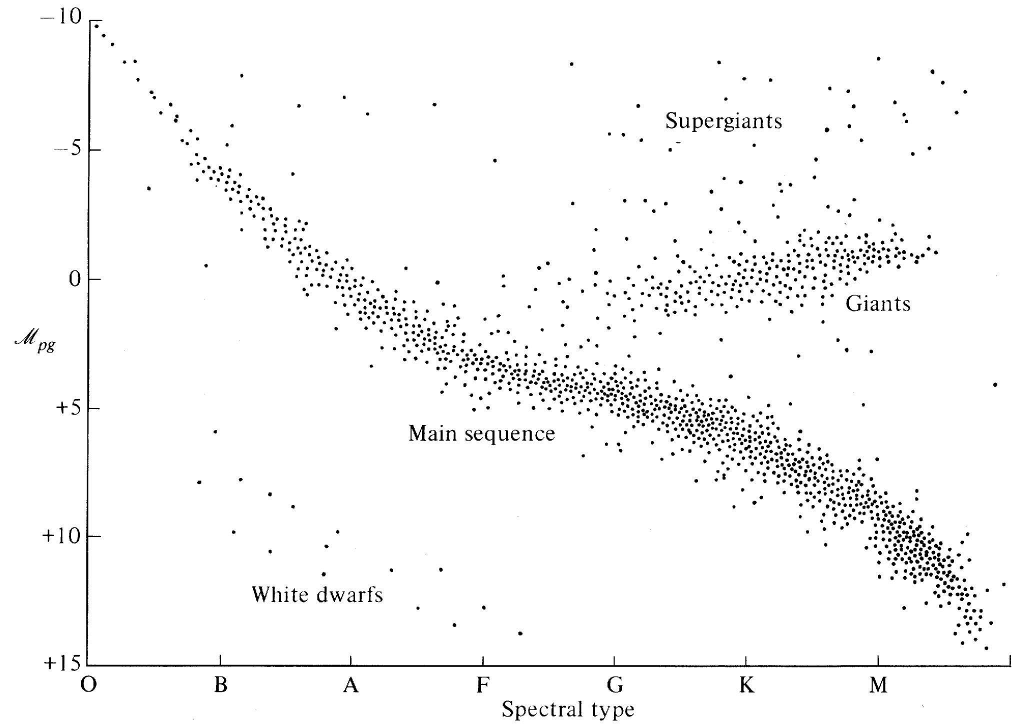


Figure 2: The Hertzsprung-Russell diagram (HRD) plotting intrinsic stellar brightness (vertical) against spectral type (horizontal). The horizontal axis follows Annie Cannon's Harvard classification which you just rediscovered. Stars to the right have red appearance, to the left they are blue. The stellar brightness is measured as "photographic magnitude", an inverted logarithmic scale for the amount of light received from different stars if they were all placed at the same distance.

The diagram is not filled randomly, but the stellar samples cluster into specific locations. Most lie in a diagonal band called the "main sequence" and a "giant branch" jutting out to the upper right, with a few scattered supergiants and white dwarfs. The star-by-star point density in this HRD corresponds to stellar statistics: there are far more dwarfs than giants — but it is likely that many more white dwarfs exist than we observe. The multiple locations per column imply that another parameter than only the spectral type is needed to define a star, which Annie Cannon and colleagues called the "luminosity classification" I–II–III–IV–V. Stars with luminosity class V are on the main sequence while stars with lower luminosity class (higher luminosity) are increasingly above it.

The HRD is the most important diagram in astronomy, but when it was first plotted (in 1908 by Hertzsprung and in 1913 by Russell) the physical meaning of these spectral type and magnitude distributions was still unclear. Only by 1925 was it established that the place of a star in this diagram obeys the simple equation $L = 4\pi R^2\,\sigma T^4$ *with* L *the luminosity of the star,* R *its radius,* T *its surface temperature, and* $\sigma = 1.3806503 \times 10^{-23}$ $m^2\,kg\,s^{-2}\,K^{-1}$ *Boltzmann's constant. The spectral classification turned out to represent ordering in surface temperature, with the hottest stars at the left. The luminosity classification follows the gas pressure in the stellar atmosphere. The stars along the main sequence are dwarfs with relatively dense atmospheres, the giants towards the upper right have more tenuous atmospheres. The modern HRD format is to plot* $\log L$ *against* $\log T$*, with the latter scale reversed in direction to maintain the traditional shape above.*

The HRD is so important because stars follows well-defined "evolution" tracks through it after their birth. For example, the sun came down from the upper right to the main sequence about five billion years ago, will stay there (between G and K) another five billion years, will then shift upward into the giant branch, and will finally complete its life along a fast arch to the lower left to become a slowly extinguishing white dwarf.

From E. Novotny, 1973, "Introduction to stellar atmospheres and interiors", Oxford Univ. Press, New York.

Sources

J. van der Rijst & C. Zwaan, 1978, *Astrofysica*, Wolters-Noordhoff, Groningen

R.J. Rutten, *Stellar Spectra A*, `http://www.staff.science.uu.nl/~rutte101/rrweb/rjr-edu/exercises/ssa`

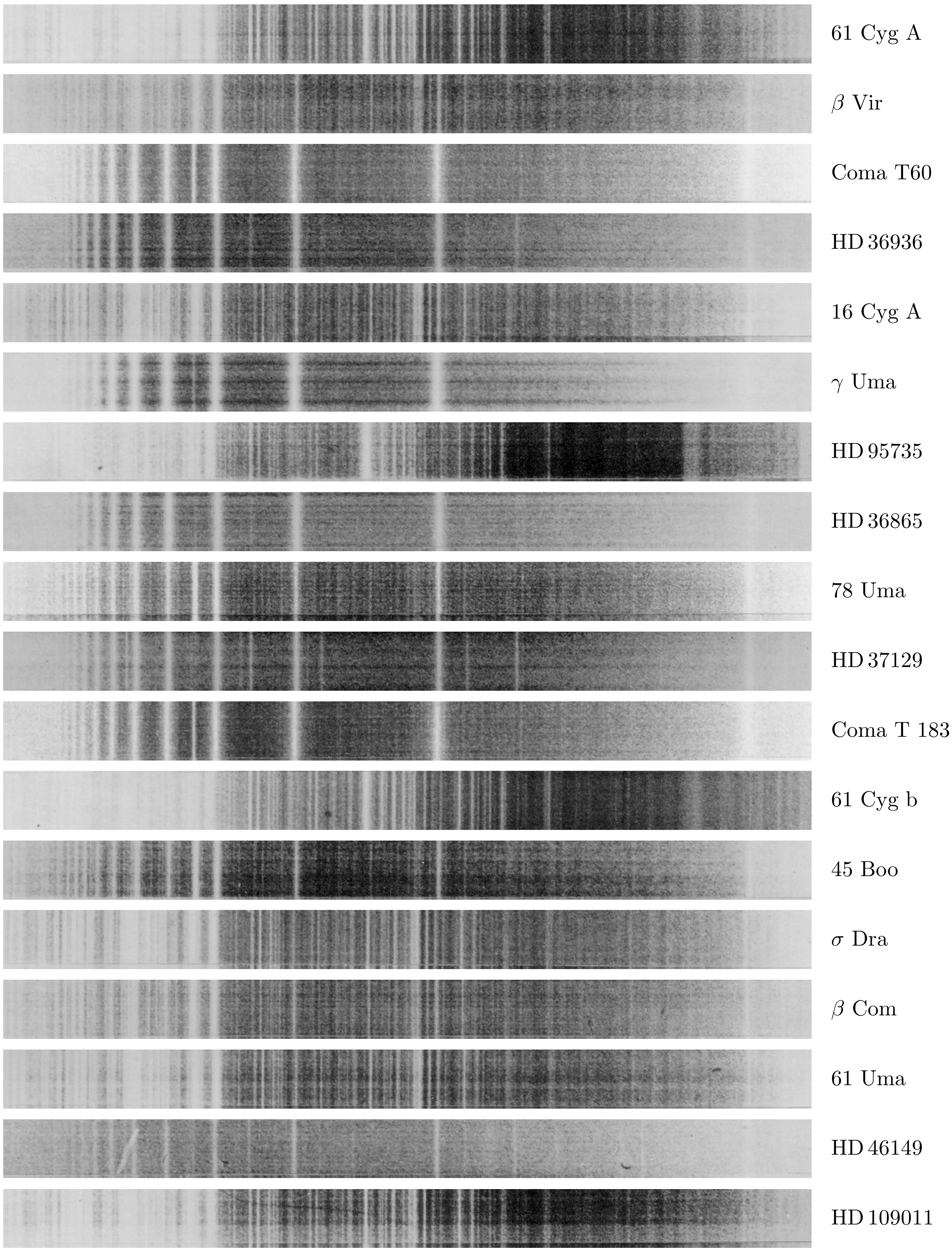


Figure 3: Stellar spectrograms covering the violet to blue part of the spectrum. The wavelength increases towards the right. The regular sequence of wide lines becoming denser to the left (e.g., γ Uma) are due to hydrogen (Balmer sequence). Taken "An atlas of low-dipsersion grating stellar spectra" by H.A. Abt, A.B. Meinel, W.W. Morgan and J.W. Tapscott (1968). After Van der Rijst & Zwaan (1978).

2 Saturn and its rings

In this assignment you study Saturn and its rings through the Doppler effect. Galileo was the first to recognize the moons of Jupiter as heavenly bodies not revolving around Earth or Sun. Christiaan Huygens was the first to recognize the ring of Saturn as non-spherical body:

> *"she has a thin flat ring around her, which touches her nowhere and is tilted with respect to the ecliptic. [...] I have to add something here to meet the criticism from those who think it strange and irrational that I assign a shape to a heavenly body of a type that has not been seen before, whereas it is believed certain and a law of nature that only a spherical shape is suited. They must realize that I don't come up with his notion of my own accord or out of fantasy, but that I see that ring clearly with my eyes."*
>
> *Chr. Huygens, 1659, "Systema Saturnium"*

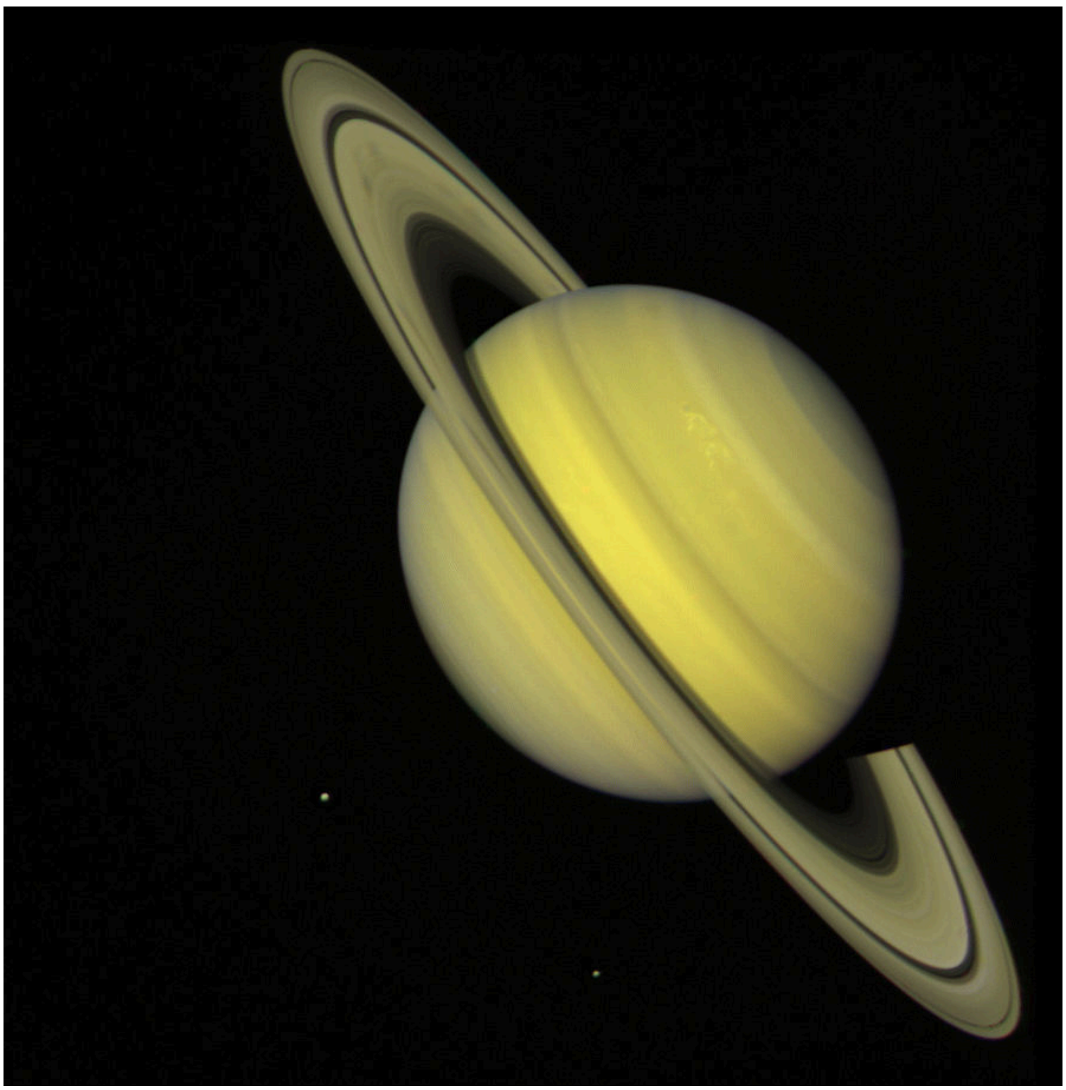


Figure 4: Saturn, photographed by Voyager 2 in 1981.

2.1 Saturn spectrogram

Figure 5 is a spectrogram taken when Saturn was in opposition: opposite to the Sun as viewed from the Earth, at its smallest distance from the Earth. The two planets approached each other with 2.85 km/s relative motion along the line of sight.

The entrance slit of the spectrograph was placed over the center of the apparent planetary disk in the telescope image, as sketched in Figure 6. The wide central band is the spectrum of the planet, the two adjacent bands come from the rings. The inner ring part, closer to the planet (ring B), is brighter than the outer part (ring A). In between lies the dark Cassini division. The fine structure of the rings is not resolved. More information: `http://saturn.jpl.nasa.gov`, `http://pds-rings.seti.org/saturn`.

Figure 5 has wavelength vertical, the location along the slit horizontal. The four specified wavelengths are given in Ångstrom (1 Å = 10^{-10} m) and hold for the white markers in the margins. These are emission lines from neon in a calibration source illuminating the ends of the slit. This source was mounted stably in front of the spectrograph. The spectral lines from Saturn and its rings clearly display Dopplershifts; the neon calibration spectra therefore serve to fix the stationary wavelength scale.

Most lines in Saturn's spectrum are not due to the planet but to the Sun. These are also present in the sunlight that reaches us on Earth, for example in the spectrum of the sky, a cloud, or your nose in daylight.

The lines in Saturn' spectrum in Figure 5 are slanted due to the Doppler effect. Saturn's rotation shifts them to the red on the side rotating towards us, to the blue on the other half. The largest shifts occur at the limbs which have the largest line-of-sight velocity. Not all lines in Figure 5 are slanted: near $\lambda = 6280$ Å is a group of non-tilted oxygen lines.

- Where are these oxygen lines formed?
- The Dopplershifts at the limbs correspond to twice the velocity at which the limb rotates towards the observer or away from the observer. Why?
- What tilt will be shown by spectral lines that originate in Saturn's atmosphere (such as methane lines in the infrared)?
- Explain with a sketch of the equatorial plane through Saturn that the tilted lines are straight across the apparent disk which Saturn shows on our sky and was projected on the spectrometer slit. Call the "viewing angle" between the line of sight towards Saturn and the normal to its surface θ. Show that $\theta = 1$ at the center of the apparent disk, $\theta = 0$ at the limbs. Then show that, along the equator, the rotational velocity component along the line of sight varies as $v_{\rm rot} \sin\theta$ while the viewing location along the radius of the apparent disk varies as $R_{\rm Saturn} \sin\theta$.

2.2 Saturn's rotation

- Measure the wavelength dispersion in Figure 5 in Å/mm and the angular scale across the image in arcsec/mm. At the time of this observation Saturn had an angular diameter of 18.5 arcsec[1]. The dispersion may be assumed constant along the spectrogram because the spectrograph used a grating, not prisms.

[1] Angular sizes are measured as size on our sky. A full circle is divided in 360 degrees (symbol °), each degree in 60 minutes of arc (arcmin, symbol ′), each arcmin in 60 seconds of arc (arcsec, symbol ″). Since a full circle is 2π radians, one radian corresponds to $360 \times 60 \times 60/2\pi = 206264.8''$. In sports time minutes and seconds are often also noted as xx' and xx'', but astronomers use $xx^{\rm m}$ and $xx^{\rm s}$.

Figure 5: Parts of a Saturn spectrogram taken at Lick Observatory by H. Spinrad and L. Giver on 19 August 1964. From O. Gingerich, Sky & Telescope 28, 278, 1964.

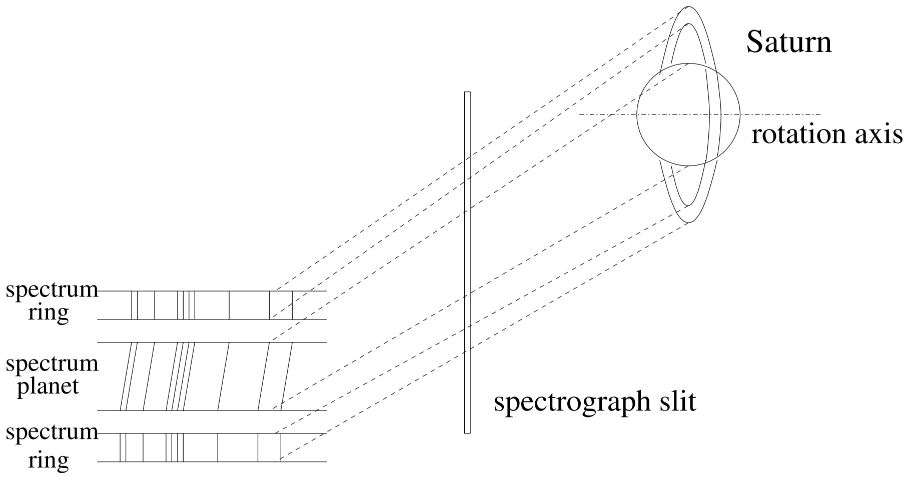


Figure 6: Sketch of Saturn spectrogram taking. The spectrograph slit sampled a diameter cut through Saturn and its rings. The light admitted through it to the spectrograph is dispersed so that each pixel along the slit has its own spectrum.

- Measure the Dopplershift at Saturn's limbs for a number of lines and take the average.
- Calculate Saturn's rotation velocity with the Dopplershift formula

$$\frac{\Delta\lambda}{\lambda} = \frac{v}{c} \tag{1}$$

  with c the speed of light (299792.458 km/s).
- Saturn's actual rotation speed is about 10 km/s. Your value may be two or four times larger. If so, explain and correct the difference.
- Calculate Saturn's radius from its rotation period of $10^{\mathrm{h}}\ 14^{\mathrm{m}}$.
- Figure 5 shows that the spectral lines of the rings do not show continuations of the planetary tilts. The slope is even reversed. Can you explain this difference?
- Explain why the lines are straight across the planet disk. Are they also straight across the limbs?
- Determine the mean rotation speed of the rings by similarly measuring wavelength shifts.

2.3 Saturn's mass

One can determine the mass of an object if it attracts another gravitationally. The mass of Saturn can therefore be determined using its rings. Do that assuming that the rocks in the rings are free particles with orbits obeying:

$$v = \sqrt{\frac{GM}{r}}, \tag{2}$$

with $G = 6.67 \times 10^{-11}\ \mathrm{N\,m^2\,kg^{-2}}$ the gravitational constant, M the mass of the planet, and r the radius of the circular orbit of a free mass-less particle orbiting the planet. This equation follows from Kepler's third law (derivation on the next page).

- Determine the mean ring diameter from Figure 5 given Saturn's radius R = 60400 km. This gives you r in (2); v you have determined already. Calculate Saturn's mass M.
- Sketch the continuation of the spectral lines if there would be inner rings consisting of small rocks with Keplerian orbits all the way down to Saturn's surface.
- Compare your sketch to the one in Figure 1 of J.E. Keeler who discussed such observations in the very first year of the Astrophysical Journal to confirm the small-body composition of the rings (he called it "meteoritic constitution"). The paper is available at ADS (bibcode 1895ApJ.....1..416K).
- Discuss whether low-orbit Earth satellites, atmospheric clouds and cannon balls have Keplerian orbits.

Sources

O. Gingerich, 1964, Sky & Telescope 28, 278

M.G.J. Minnaert, 1969, *Practical work in elementary astronomy*, Reidel, Dordrecht

J. Kleczek, 1987, *Exercises in astronomy*, Reidel, Dordrecht

Kepler's three laws

During 1609 – 1619 Johannes Kepler formulated his three laws for the motion of planets around the Sun:

1. *elliptic orbits:* planets move along ellipses having the Sun in one focus;
2. *equal areas:* the line from a planet to the Sun sweeps out equal areas in equal times while the planet travels along the ellipse;
3. *harmonic ratios:* the ratio of the squares of the orbital periods for two planets equals the ratio of the cubes of their semimajor axes.

Subsequently, in 1621, Kepler showed that the orbits of the four moons of Jupiter, described in 1610 by Galileo, also obey the third law. Newton rewrote it in 1684 in general form:

$$P^2 = \frac{4\pi^2\, a^3}{G\,(M_1 + M_2)},$$

with P the orbital period, a the semimajor axis, $G = 6.67{\times}10^{-11}$ N m^2 kg^{-2} the gravitational constant, and M_1 and M_2 the masses of the two objects. This equation holds universally for gravitational motion of one body around another, for example the companions in binary stars. In the case of a planet orbiting the Sun the planet mass M is negligible: $M \ll M_\odot$. The same holds for the rocks making up Saturn's rings so that their orbital period is

$$P^2 = \frac{4\pi^2\, a^3}{G\,M}$$

with M the mass of Saturn. For circular motion with radius $a = r$ and orbital velocity v the orbital period is also given by

$$P = 2\pi r\, v$$

and the combination of these two yields Eq. (2):

$$v = \sqrt{\frac{GM}{r}}.$$

The Newtonian derivation is to equate the centripetal force F_c describing the radial acceleration in a circular orbit $a_\mathrm{c} = v^2/r$ and the gravitational attraction F_g:

$$F_\mathrm{c} = M_1\,\frac{v^2}{r} \qquad\qquad F_\mathrm{g} = G\,\frac{M_1 M_2}{r^2}.$$

More detail at `http://hyperphysics.phy-astr.gsu.edu/hbase/hph.html`.

3 Mapping the Milky Way

In this assignment you determine the near-by spiral structure of our host galaxy, the Milky Way, following the pioneering analysis published by H.C. van de Hulst, C.A. Muller and J.H. Oort in 1954.

3.1 Galaxies

A galaxy is a concentration of many stars and large "interstellar" clouds of gas and dust. All stars that you see at night belong to our own galaxy, the Milky Way. Just like most others (about 80%) it is a spiral galaxy comparable to the examples in Figure 7, having spiral arms in a flat disk and a more spherical central bulge. The center contains a large black hole.

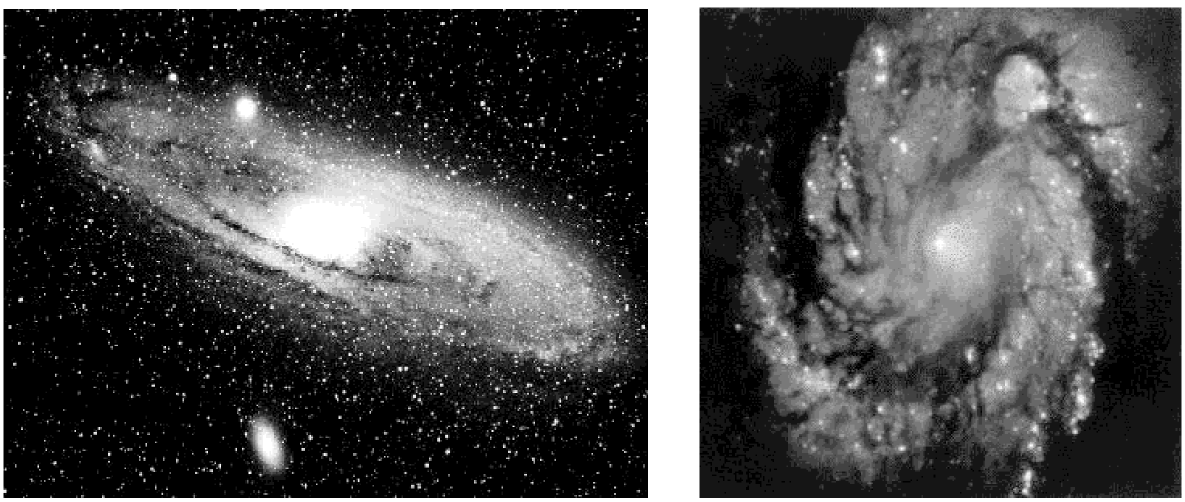


Figure 7: Left: M31 (Andromeda nebula), the only galaxy outside our own that is visible to the naked eye (apart from the Magellanic Clouds, our small satellite galaxies). Right: M100, image from the Hubble Space Telescope.

Andromeda (left in Figure 7) is our neighbor. Its mass $M_{\mathrm{And}} = 3 \times 10^{11} M_{\odot}$ (solar masses) and a radius of about 40 kpc.

- How many stars are there in the Andromeda galaxy?
- What is the diameter of the Andromeda nebula in km and in astronomical units (see Table 1 on page 15)?

The Andromeda nebula is not only our closest neighbor but is also similar to the Milky Way. Because the solar system lies within the galactic plane we cannot directly appreciate the spiral structure; that needs viewing from the side as for M100 in Figure 7. The M100 image also shows many dark clouds. Our galaxy has them too; for example, they obscure the center of our galaxy (for viewers on the southern hemisphere).

Gas clouds emit radiation with a specific wavelength of 21 cm in the radio domain of the electromagnetic spectrum. This radiation is much less blocked by interstellar clouds than visible light. You use this property here to determine the location of interstellar clouds in the nearby outer arms of our galaxy.

3.2 The 21-cm line

Hydrogen is the most abundant element in the universe. Hydrogen is also the principal constituent of the gas clouds studied here. Hydrogen atoms consist of a single proton acting as atomic nucleus and a single orbiting electron as outer shell. The normal spectral lines of hydrogen are caused by jumps of the electron between different permitted orbitals, absorbing or emitting a photon when being excited or in de-excitation. These lines lie in the ultraviolet (Lyman sequence from the ground state), the visible (Balmer lines from the first excited state), and the infrared (Paschen, Brackett, etc.)

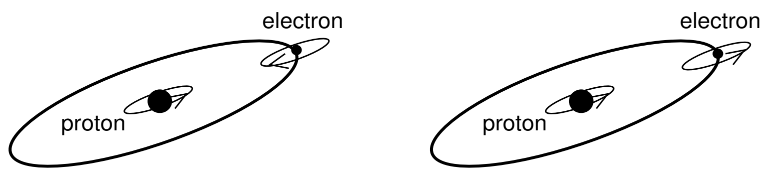


Figure 8: Left: hydrogen atom in the ground state. Right: excited hydrogen atom. Note that sketching spins as rotation is a simplification, as is sketching protons and electrons as little spheres with circular electron orbits. The orbitals are set by Coulomb interaction, the spin interactions are magnetic and produce dipole fields, as tiny bar magnets.

The 21-cm radio line is a special one. It is not due to electron jumps between different orbits but to a fine-structure change of the electron spin. There are two spin orientations: parallel and opposite between electron and proton (Figure 8), with a small energy difference between the two. The one with opposite spins is the lowest ("ground") state in energy. A hydrogen particle in that state stays so forever if left undisturbed, but it can be excited, usually through a collision with another hydrogen atom, into the higher-energy parallel-spin state. If it then remains undisturbed it will naturally ("spontaneously") revert back to the lower state, but that takes very long: the mean lifetime in the upper state is 10^7 year in the absence of any further interactions. When it falls back it emits a 21-cm photon in a random direction. This is a very rare occurrence per hydrogen atom, but the massive interstellar galactic clouds contain enough hydrogen to make this radiation nevertheless detectable[2].

- The fine-structure energy difference is $\Delta E = 9.409 \times 10^{-25}$ J. Show that this corresponds to wavelength $\lambda = 21$ cm for the emitted photon.

3.3 The rotation of the Milky Way

The stars and clouds in the outer Milky Way move around the galactic center in a fairly thin plane. We assume here that their orbits are circular and obey Kepler's laws (as the rocks in the rings of Saturn). First estimate the mass of the galactic center.

- The Sun is at distance R_0 from the galactic center and orbits it with velocity $v_\odot$. See Table 1 on page 15 for the numerical values. Calculate the Sun's orbital period.
- How many times has the Sun gone round?
- Use Eq. (2) on page 10 to calculate the mass of the galactic center, in kg and in solar masses $M_\odot$. This is formally the total mass of the Milky Way out to $R_0 = 10$ kpc, but it is reasonable to assume

[2]As predicted already in 1944 by H.C. van de Hulst, at that time astronomy student in Utrecht. His estimate was initiated by J.H. Oort's question whether there exist any spectral lines at radio wavelengths that might serve as galactic Doppler measurers. The answer was yes. See the reminiscences by Van de Hulst about this exciting prediction on page 21.

pi	π	=	3.14159
velocity of light	c	=	2.99792×10^{8} m s^{-1}
gravitational constant	G	=	6.67259×10^{-11} m^{3} kg^{-1} s^{-2}
Planck's constant	h	=	6.62607×10^{-34} J s
mass of the Sun	$M_{\odot}$	=	1.9891×10^{30} kg
radius of the Sun	$R_{\odot}$	=	6.9599×10^{8} m
astronomical unit (distance Sun–Earth)	AE	=	1.49598×10^{11} m
lightyear	ly	=	9.4605×10^{15} m
parsec	pc	=	3.2616 ly = 3.0857×10^{16} m
distance Sun — Galactic Center	R_0	=	10 kpc
orbital speed of the Sun around the Galactic Center	$v_{\odot}$	=	250 km s^{-1}

Table 1: Constants

that most of this mass is concentrated close to the center.

3.4 Radial velocity between the Sun and a cloud

The lefthand sketch in Figure 9 defines a few quantities needed to use the apparent Dopplershift of 21-cm radiation from an interstellar cloud.

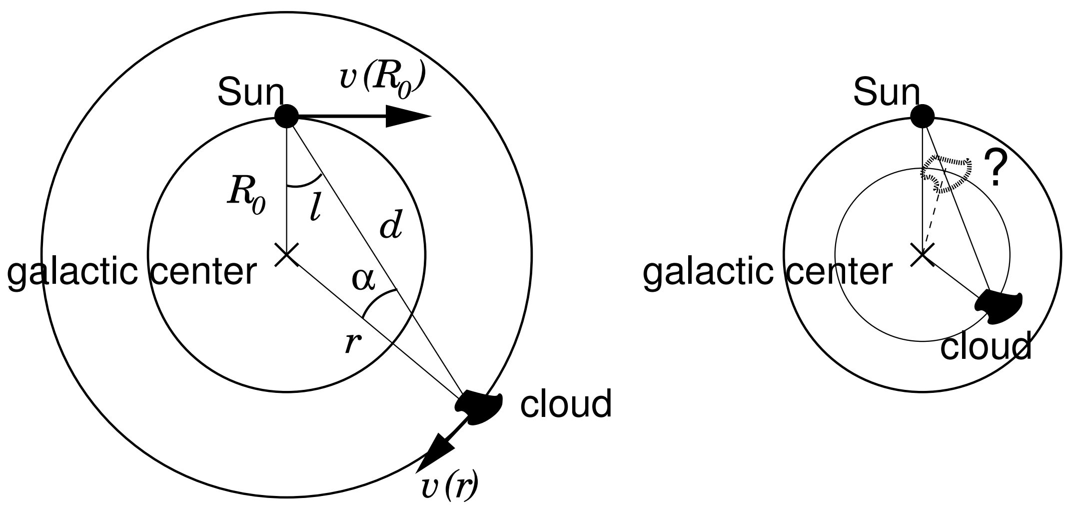


Figure 9: Geometry in measuring Dopplershifts of galactic clouds. Left: definition of various quantities. Right: knowledge of l and r leaves ambiguity in the location of clouds that orbit within the solar orbit.

The Sun is at distance R_0 from the galactic center. The direction from the Sun to the cloud is given by the galactic longitude angle l with $l = 0$ in the direction to the center and $l = 180°$ the opposite point on our sky. A cloud at distance d from the Sun and distance r from the center has orbital velocity $v(r)$. Its component along the line of sight from the Sun is $v(r) \sin \alpha$. The Sun has orbital speed $v(R_0) = v_{\odot}$ with component $v(R_0) \sin l$ along the line of sight to the cloud. The radial velocity[3] of the cloud with respect

[3]Dopplershift measurement yields velocities that are called "radial" because they sample only the motion component along the line of sight, away from us or towards us. The perpendicular component is called "transversal" or more often "proper motion" as being the apparent movement on our sky. Radial velocities are measured in km/s, proper motions in angular units such as ″/year.

to the Sun is:

$$v_{\rm rad} = v(r)\sin\alpha - v(R_0)\sin l. \quad (3)$$

- Check Eq. (3).
- Radial velocities are taken positive for redshift (towards longer wavelength). Show that the cloud in the lefthand sketch shows blueshift. Is it falling towards us?

The treatment on page 20 shows that combination of Eq. (3) with Eq. (2) on page 10 yields:

$$\frac{v_{\rm rad}}{\sin l} = v(R_0)\left[\left(\frac{R_0}{r}\right)^{3/2} - 1\right]. \quad (4)$$

This equation gives a relation between the two measurements per cloud (galactic longitude l and radial velocity $v_{\rm rad}$) and its distance r to the galactic center. It is shown graphically in Figure 10.

The righthand sketch in Figure 9 illustrates that measuring l and r leaves ambiguity in cloud location. We therefore exclude clouds looted within the solar orbit by inspecting only those with:

$$v_{\rm rad} < 0 \;\text{ for }\; 0 < l < 180^\circ \qquad\qquad v_{\rm rad} > 0 \;\text{ for }\; 180 < l < 360^\circ. \quad (5)$$

- Check that these criteria work properly.
- The ambiguity can often be solved by measuring the extent of the cloud in galactic latitude b. What assumptions are needed?

3.5 Radio spectra of Milky Way clouds

Figure 11 contains spectral profiles of the 21-cm line collected with a former 7.5-m radar antenna at Kootwijk in the Netherlands, measured in 54 directions within the galactic plane. The galactic longitudes l^I (numbers in degrees) refer to an older definition having the galactic center at $l^I = -33^\circ$; you need to add 33° to the values in Figure 11 to obtain the modern longitudes l. The profiles show the radiation intensity as function of redshift expressed in km/s. The vertical lines mark the rest wavelengths with $\Delta\lambda = 0$. The scale is at the bottom.

Each peak indicates a concentration of hydrogen gas (cloud or spiral arm) along the line of sight. Assume that these objects are transparent for 21-cm radiation so that the contributions of multiple clouds along the line of sight add up without blocking.

- Why are the spectra for $l^I = 327^\circ$ and $l^I = 147 - 150^\circ$ nearly symmetric and without shifted peaks?
- Measure the radial velocity of all peaks with negative redshifts from $l^I = 327^\circ$ to $l^I = 145^\circ$. Note them in a tabular scheme as suggested in Table 2 on page 17.
- Measure the radial velocity for all peaks with positive redshift for the remaining directions $l^I = 322^\circ$ and $150 < l^I < 220^\circ$.

3.6 Milky Way map

As said, Figure 10 is a plot of Eq. (4). Per combination of $v_{\rm rad}$ and $l = l^I + 33^\circ$ it furnishes the corresponding distance to the galactic center.

- Determine the distances $r(l)$ for all measured peaks using Figure 10.
- Plot the locations of all these gas concentrations in a (l,r) chart. If you do this manually on graph paper then put the galactic center at the center of your plot, the Sun 5 cm above it. In older days one would use polar graph paper for l and calipers for r (with the Sun at the center, the galactic center 5 cm below). Of course, no one plots graphs manually anymore – but you might do so for once and see your Milky Way grow in the process.
- Can you recognize spiral structure? The Sun is located in the Orion arm. Outside, at 2 kpc, lies the Perseus arm.
- Compare your map with the one in the article of Van de Hulst, Muller and Oort, available at ADS (bibcode 1954BAN....12..117V).

3.7 Dark epilogue

Later studies, initially by Oort and collaborators at Leiden and subsequently especially at Groningen, showed that the assumption of Kepler orbits with $v \propto 1/\sqrt{r}$ as in Eq. (2) on page 10 is incorrect. Galaxies tend to rotate more stiffly in their inner part whereas the rotation of the outer part is usually more or less constant with r. This flat shape of the outer part of galactic rotation curves was the major original motivation to invoke the existence of dark matter in invisible halos around galaxies. Nowadays the evidence for dark matter comes primarily from galaxy-cluster dynamics.

The rotation curve of our own galaxy remains relatively badly known due to the obscuration of the stellar population which we suffer by being located in the galactic plane. The obscuration is by truly dark normal matter (not a dark horse).

Sources

H.C. van de Hulst, C.A. Muller, J.H. Oort, Bull. Astron. Inst. Netherlands, 12, 117, 1954
M.G.J. Minnaert, 1969, *Practical work in elementary astronomy*, Reidel, Dordrecht
O. Gingerich, 1984, Sky & Telescope 68, 10
J. Kleczek, 1987, *Exercises in astronomy*, Reidel, Dordrecht

Table 2: Measuring scheme 21-cm profiles Milky Way

l^I	l	$\sin l$	$v_{\rm rad}$ [mm]	$v_{\rm rad}$ [km/s]	$v_{\rm rad}/\sin l$	r

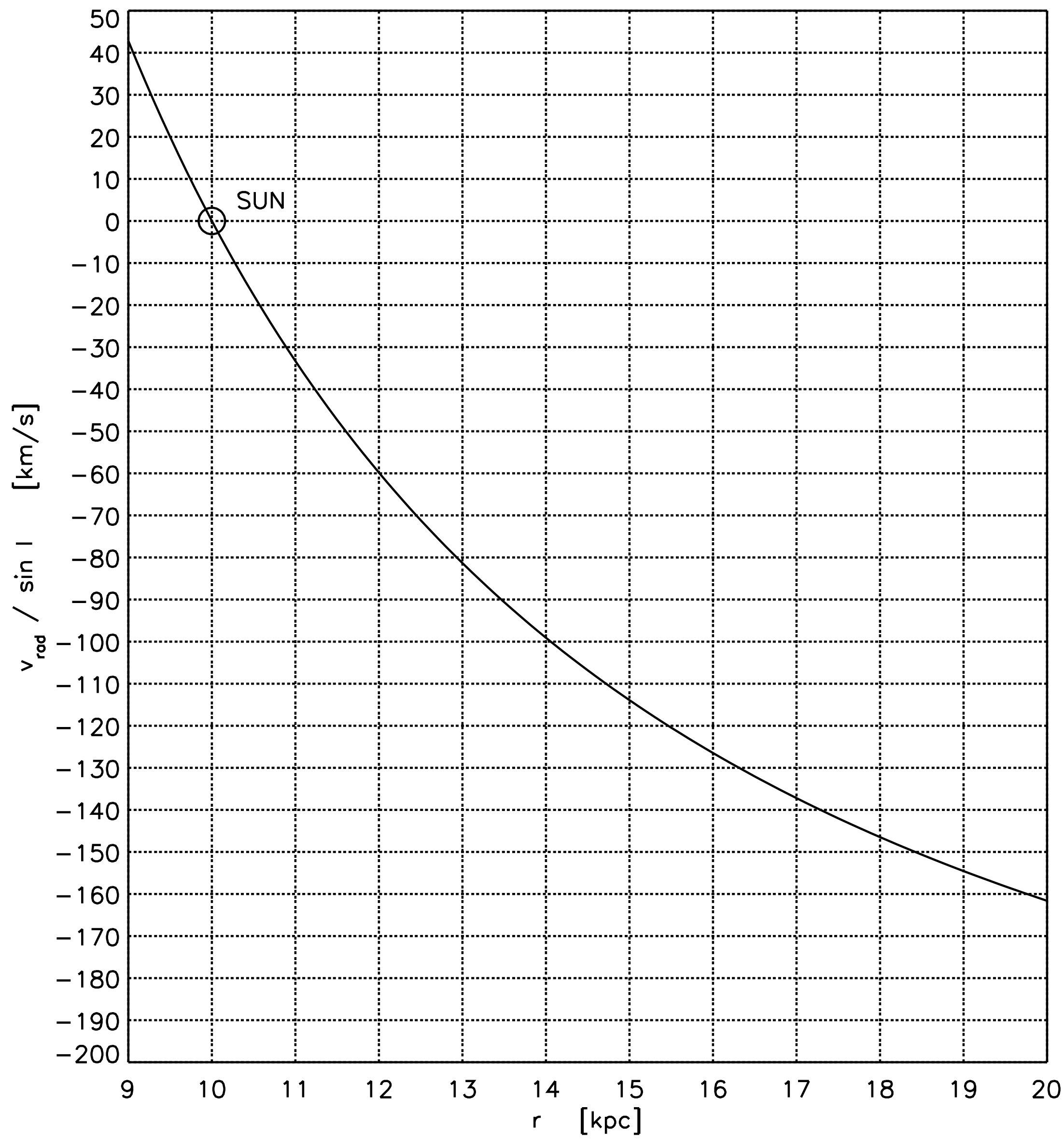


Figure 10: Graph of Eq. (4).

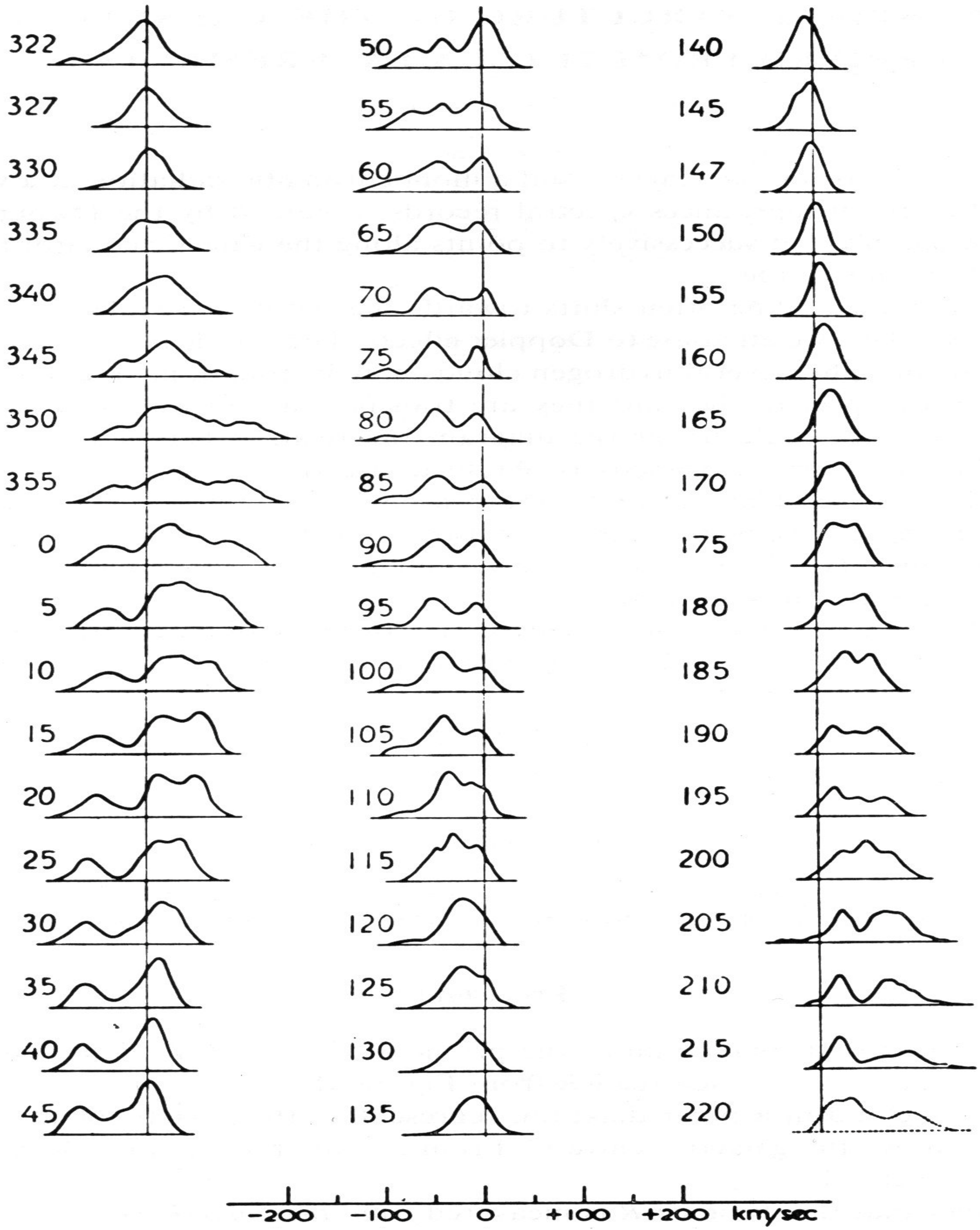
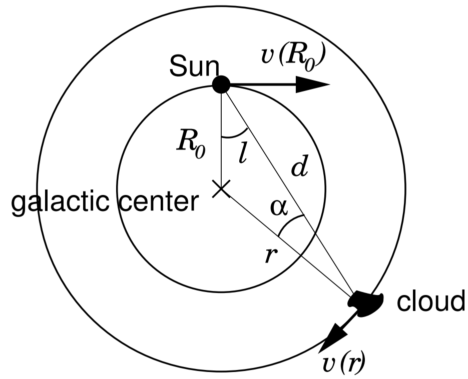


Figure 11: Galactic spectra, from H.C. van de Hulst, C.A. Muller, J.H. Oort, Bull. Astron. Inst. Netherlands, 12, 117, 1954. Vertical: radiation intensity. Horizontal: wavelength plotted as redshift in km/s according to the scale below. The numbers specify galactic longitude l^I in degrees, an old coordinate system with the galactic center in direction $l^I = -33°$. All measurements concern the galactic plane (galactic latitude $b = 0°$).

Derivation of Eq. (4)



The measured difference in radial velocity between the Sun and the object is given by Eq. (3):

$$v_{\rm rad} = v(r)\sin\alpha - v(R_0)\sin l.$$

By drawing a line from the galactic center perpendicular to the line of sight from the Sun to the object one finds:

$$R_0 \sin l = r \sin\alpha,$$

or

$$\sin\alpha = \frac{R_0}{r}\sin l,$$

so that the radial velocity of the cloud with respect to the Sun is:

$$v_{\rm rad} = R_0 \ \sin l \left[\frac{v(r)}{r} - \frac{v(R_0)}{R_0}\right].$$

For known radial velocity $v_{\rm rad}$ the ratio $v(r)/r$ remains as the unknown:

$$\frac{v(r)}{r} = \frac{v_{\rm rad}}{R_0 \sin l} + \frac{v(R_0)}{R_0},$$

but by assuming Keplerian orbits this ratio and $v(R_0)/R_0$ can be rewritten with Eq. (2) on page 10 as:

$$\frac{v(r)}{r} = \sqrt{\frac{GM_r}{r^3}} \qquad \frac{v(R_0)}{R_0} = \sqrt{\frac{GM_{R_0}}{R_0^3}}$$

with M_r the mass of the Milky Way within radius r. Some algebra and the assumption $M_r \approx M_{R_0}$ then yield Eq. (4):

$$\frac{v_{\rm rad}}{\sin l} = v(R_0)\left[\left(\frac{R_0}{r}\right)^{3/2} - 1\right].$$

This relation is plotted in Figure 10 on page 18.

Reminiscences by H.C. van de Hulst

Excerpt from Annual Review of Astrophysics & Astronomy, 1998, 36, 1
ADS (bibcode 1998ARA&A..36....1V)

In mid-1944, still in the dark days of war, Oort was preparing a national colloquium on what would later become known as radio astronomy. But at the time that term did not yet exist; Reber had used the term "cosmic static". Oort had a very fine nose for what might become important, and he insisted that this topic be fully reviewed. Dr. Bakker (from Philips Research) was asked to explain the instrumental side, and, knowing my interest in the interstellar gas, Oort asked me to review the theoretical explanation suggested by Henyey and Keenan. So overnight, I shifted fields and became an expert in the new field of radio astronomy. Another remark by Oort had far-reaching consequences. Oort said: "The attractive property of these radio waves is that they propagate through the entire Galaxy without attenuation. If there were a spectral line which we could measure, the Dopplershift could be used to map the motions in the Galaxy." Only much later did I realize that this remark was a most natural extension of the work reported in Oort's own thesis of about 20 years before.

So I started to study spectral lines and spent several months systematically exploring all kinds of possibilities. Incidentally, I made a substitution error in estimating the broadening of the "recombination lines" by the Stark effect, arriving at the erroneous conclusion that they would all be effaced by broadening. Later, one afternoon, my systematic exploration brought me to the hyperfine structure line arising from a turnover of the electron spin in the field of the proton, all in the ground state of atomic hydrogen. By putting the magnetic moment of the proton – recently measured by Kopferman – into a theoretical formula derived by Fermi, I concluded that an interesting line should exist with a wavelength of 21 cm. The intensity of this line seemed very uncertain. But by assuming that the magnetic dipole strength would be of the order 1 and making a somewhat liberal guess about future instrumental development, I estimated that some time in the future, this line might become observable in the Galaxy. The dramatic history is that it took another seven years before, in 1951, Ewen & Purcell first detected this line, followed rapidly by confirmations in Holland and in Australia. After this exciting discovery, Oort had the wisdom to permit Muller, the chief engineer at our Kootwijk receiving station, ample time for constructing improved receiving equipment. As a result, the 53 line profiles, measured all along the northern Galactic equator and presented in 1953, immediately became a "classical" result that was widely cited. Interpreting the maxima and minima, we could distinguish three new spiral arms in our Galaxy.

4 The Hubble constant

4.1 The expanding universe

In 1929 Edwin P. Hubble wrote his classic paper establishing a linear relationship between the redward line-of-sight Dopplershift shown by nearby galaxies and their distance. Figure 12 is his original figure plotting this relation in terms of corresponding recession velocity.

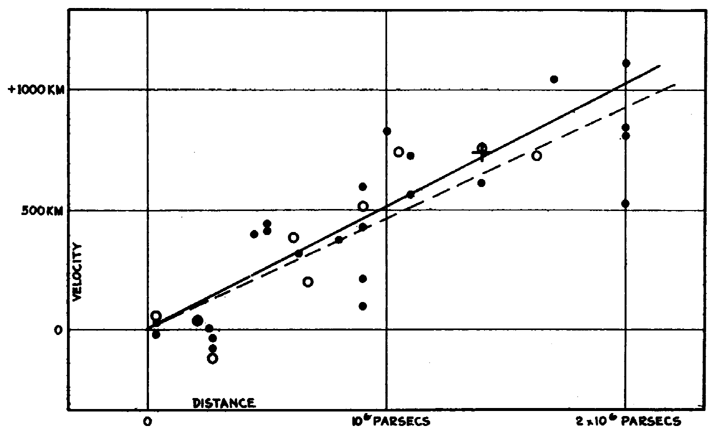


FIGURE 1

Velocity-Distance Relation among Extra-Galactic Nebulae.

Radial velocities, corrected for solar motion, are plotted against distances estimated from involved stars and mean luminosities of nebulae in a cluster. The black discs and full line represent the solution for solar motion using the nebulae individually; the circles and broken line represent the solution combining the nebulae into groups; the cross represents the mean velocity corresponding to the mean distance of 22 nebulae whose distances could not be estimated individually.

Figure 12: First plot of the Hubble relation, published by Hubble in 1929 in the Proceedings of the National Academy of Sciences of the United States of America, 15, 68, available at ADS (bibcode 1929PNAS...15..168H).

It is now called the Hubble law:

$$v = H_0 D \tag{6}$$

with v the recession velocity in km/s, D the distance in Mpc (megaparsec), and H_0 the Hubble constant in km s^{-1} Mpc^{-1}. The latter is the measure for the expansion speed of the Universe. The index 0 distinguishes the constant (here and now) from the time-dependent Hubble variable H in the Friedmann equations of cosmology. Hubble used galaxies that are sufficiently close that the time of observation equals the time at which the radiation left and the distance measured now is still the distance then.

The cosmological deceleration parameter q is defined through the time-dependent Hubble parameter as:

$$q = -\frac{1}{H^2}\left(\frac{\mathrm{d}H}{\mathrm{d}t} + H^2\right). \tag{7}$$

It is zero when $H = 1/t$ with t the age of the universe since the Big Bang; $1/H_0$ is the Hubble time. In standard cosmology $H_0 = 70.9$ km s^{-1} Mpc^{-1} and $1/H_0 = 4.35 \times 10^{17}$ s or 13.8×10^9 years. Although q was long believed to be 1/2, studies of supernovae suggest that the expansion of our universe actually accelerates. The current best estimate for the age of the universe, from WMAP (Wilkinson Microwave Anisotropy Probe, `http://map.gsfc.nasa.gov/`), is $(13.7 \pm 0.2) \times 10^9$ years.

In this assignment you re-enact Hubble's discovery, measure the Hubble constant from five galaxies, and use its value to determine the distance of a quasar.

4.2 Distances of five galaxies

Figure 14 contains images and spectra of five galaxies at different distances. They are of the same elliptical type so that you may assume that they are intrinsically similar and that their apparent angular sizes differ primarily through difference in distance. The sketch in Figure 13 defines the angular geometry. A galaxy at distance D with apparent diameter S on our sky (perpendicular to the line of sight) is observed as having angular diameter d. The radius has

$$\tan d/2 = \frac{S/2}{D} \qquad (8)$$

but since $\tan\alpha \approx \alpha$ for small angle α when measured in radians (check with your calculator) you can rewrite this as

$$D = \frac{S}{d}. \qquad (9)$$

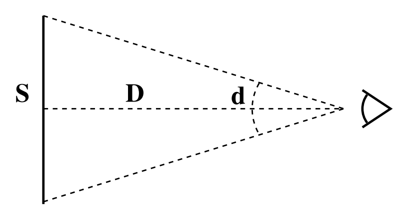


Figure 13: Distance to a galaxy.

- Measure the diameter of the five galaxies in mm. Average the short and long diameters for non-spherical cases.
- Convert the diameters into arcsec with the scale specification at bottom left.
- Convert the diameters into radians (1 rad = 206265″, see footnote on page 8).
- You now have values of d in Eq. (9). The typical size of this type of galaxy is $S \approx 0.03$ Mpc. Now compute the five distances D in Mpc.

4.3 Redshifts of the five galaxies

The righthand part of Figure 14 shows spectra of the galaxies, with helium calibration spectra above and below. Each spectrum contains, next to other lines, a prominent pair of dark ines which are Ca II H & K of once-ionized calcium atoms. For cool stars such as the Sun these are the strongest lines in the visible part of the spectrum. In the light of a whole galaxy they appear as the sum over billions of such cool stars.

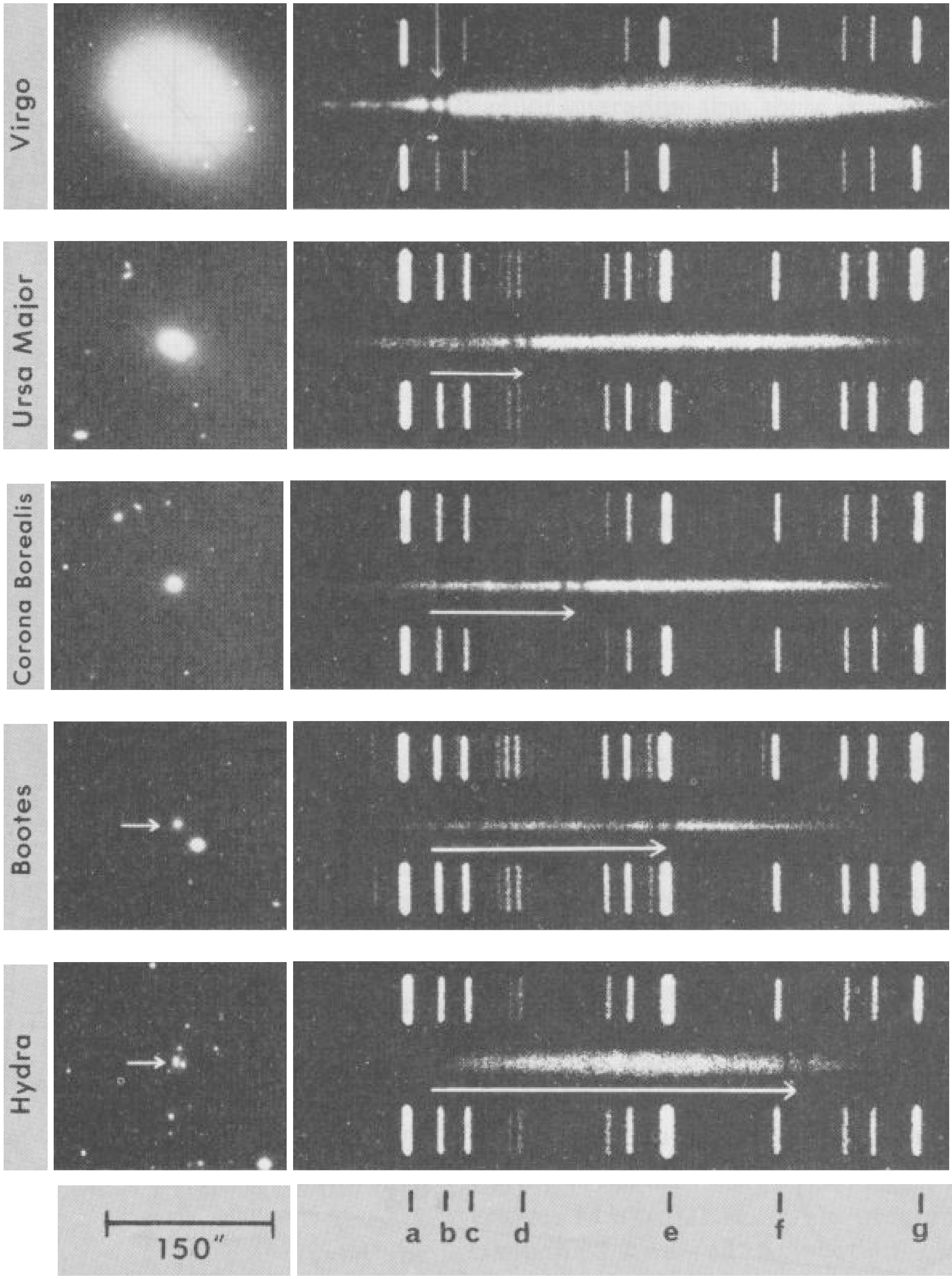


Figure 14: Images and spectra of five galaxies taken at the Hale Observatory. The thin arrow in the first spectrum identifies the H & K lines (K is the lefthand one, H was named by Fraunhofer). The horizontal arrows below the galactic spectra show the H & K redshifts. The marked lines in the calibration spectra are from neutral helium gas and have laboratory wavelengths: λ_a = 3888.7 Å, λ_b = 3964.7 Å, λ_c = 4026.2 Å, λ_d = 4143.8 Å, λ_e = 4471.5 Å, λ_f = 4713.1 Å *and* λ_g = 5015.7 Å. *The laboratory wavelengths of H & K are:* λ_H = 3968.5 Å, λ_K = 3933.7 Å.

From top to bottom the H & K lines shift from left to right, as indicated with white arrows. These shifts express different recession velocities via the Dopplershift formula (Eq. (1) on page 10):

$$v = c\,\frac{\Delta\lambda}{\lambda} \tag{10}$$

with c the velocity of light (299792.458 km s^{-1}) and $\Delta\lambda$ the shift in wavelength between the observed line and its laboratory value λ.

- Measure the arrow lengths and convert them to Ångstrom using the calibration spectra (1 Å = 10^{-10} m; this was the traditional unit for optical wavelengths).
- Compute the recession velocities with Eq. (10). Can you neglect the component of the solar orbital velocity around the galactic center (Table 1 on page 15)?

4.4 Estimation of the Hubble constant

- Plot your five recession velocities against the corresponding distances.
- Draw a best-fit line through the five points and determine its slope. This is your estimate of the Hubble constant and the age of the universe.

4.5 Quasar distance

Figure 15 shows the spectrum of quasar Q0002+051. The central peak is the hydrogen Lyman α line, in emission. This line is in the far ultraviolet at wavelength λ = 1216 Å in a terrestrial laboratory but is here located at λ = 3530 Å in the very near ultraviolet. There are also many absorption lines; the most prominent are numbered 1 to 25. These are also due to Lyman α transitions in hydrogen atoms.

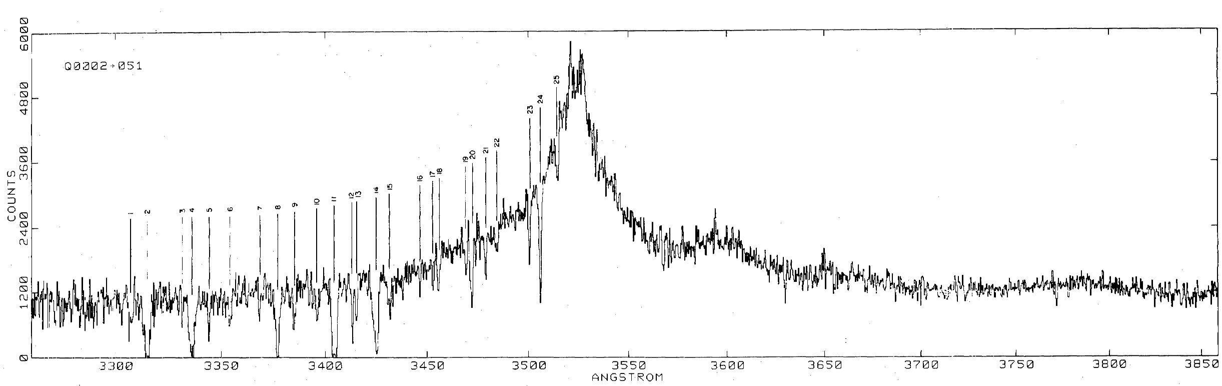


Figure 15: The spectrum van of quasar Q0002+051, from P. Young, W.L. Sargent & A. Boksenberg, 1982, available at ADS (bibcode 1982ApJ...252...10Y).

- Calculate the recession velocity of Q0002+051 using the emission peak in Eq. (10). What is wrong with the result?

Eq. (10) is an approximation valid only for $v/c \ll 1$. The complete ("relativistic") formula for the Doppler effect is:

$$z \equiv \frac{\Delta\lambda}{\lambda} = \sqrt{\frac{1 + v/c}{1 - v/c}} - 1. \tag{11}$$

The derivation in the box below shows how to get from (11) to (10) for small v/c.

- Move the 1 in Eq. (11) to the other side and take the square of both sides to rewrite it into:

$$v = c\,\frac{(z+1)^2 - 1}{(z+1)^2 + 1}. \tag{12}$$

- Determine z and the recession velocity of Q0002+051.
- Use your value of the Hubble constant to determine the distance to the quasar. This is not the most distant one; the record stands at $z = 6.4$.
- How many years did the light of this quasar take to reach us? (Conversion factors in Table 1 on page 15.)
- The numbered Lyman α absorption lines are all to the left of the peak. What causes them?

Sources

A. Evans, 1978, Sky & Telescope 55, 299

http://en.wikipedia.org/wiki/Age_of_the_universe

Derivation of the approximate Dopplershift formula (10) from (11) for $v/c \ll 1$

Assume $v/c \ll 1$. Rewrite Eq. (11) with $v/c \equiv \epsilon$ into

$$z \equiv \frac{\Delta\lambda}{\lambda} = \sqrt{\frac{1+v/c}{1-v/c}} - 1 = \sqrt{\frac{1+\epsilon}{1-\epsilon}} - 1,$$

express the square root as power

$$z = (1+\epsilon)^{\frac{1}{2}}\,(1-\epsilon)^{-\frac{1}{2}} - 1,$$

use the approximation $(1+\epsilon)^x \approx (1 + x\,\epsilon)$ for $\epsilon \ll 1$ (try on your calculator) to obtain

$$z \approx (1+\epsilon/2)\,(1+\epsilon/2) - 1 \approx (1+\epsilon+\epsilon^2/4) - 1 \approx \epsilon + \epsilon^2/4,$$

and neglect the quadratic term $\epsilon^2/4$ since it is much smaller than ϵ. The result is Eq. (10):

$$z \approx \epsilon = v/c.$$